Air Traffic Incidents Analysis with the Use of Fuzzy Sets

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Abstract. In safety, reliability as well as risk analysis and management, information often is uncertain and imprecise. The approach to air incident analysis under uncertain and imprecise information presented in our paper is inspired by the possibility theory. Notably, in such analyses these are both: static and dynamic components that have to be included. As part of this work, static analysis of a serious incident has been performed. In order to do this, probability scale which is based on fuzzy set theory has been given. The scenarios of transformation of incident into accident have been found and their fuzzy probabilities have been calculated. Finally, it has been shown that elimination of one of premises for transformation of the incident into accident significantly reduces the possibility of this transformation.

Keywords: serious incident, fuzzy probability, events tree, fuzzy inference, air traffic safety.

1 Introduction

Air communication is commonly thought as the most safe transport type. Because passenger safety is the main priority of all subjects engaged in air transport, technical, organization, procedure barriers are established in order to avoid air accidents. Sometimes these facilities fail; in most cases because of human error. To learn lessons from these failures, accidents are investigated in order to find their causes. Such investigation is usually qualitative [8].

In the paper, a quantitative analysis of serious incidents is proposed. The "serious incident" is usually a very dangerous event when some barriers against accident have failed to meet their goal. They are very important sources of knowledge about safety assurance systems in air transport. We want to estimate the probability that a given incident would transform into accident. With that kind of study at disposal, one can conclude whether safety facilities are sufficient or have to be extended. In order to evaluate this probability, estimation of safety barrier reliability has to be carried out. Unfortunately, in most cases there are no sufficient data to infer statistically about the frequency of events for the accident scenario. Unfortunately, it is highly unlikely to find that data. There are two reasons of such situation. First is that some of these events occur very rarely, and additionally, in past the events without significant consequences were not usually recorded. The second one is human factor with such

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measures that are difficult to evaluate as different reactions probabilities and error activity probability. Such measures are charged with uncertainty and subjective estimations. Only methods to obtain such knowledge are expert estimates. These estimations are not precise and not sufficient to probabilistic analysis.

In safety, reliability, and risk analysis and management, information often is uncertain and imprecise. In book [10] three approaches to reliability and safety with uncertain and imprecise information are presented: probability and statistics, fuzzy set theory, possibility theory (inspired by the above).

In paper [1] the following approaches for representation of uncertainty are listed: probability, imprecise (interval) probability, probability bound analysis, possibility theory (foundations: probability, statistics, fuzzy sets), Dempster-Shafer evidence theory.

The approach to air incident analysis presented in our paper is inspired by the possibility theory.

In air incident analysis both types of components have to be included: static and dynamic. Static analysis can be executed by means of fault trees with fuzzy probabilities [16, 17] and event trees with fuzzy probabilities [7]. Fuzzy probability is called possibility. The Dynamic analysis is executed in the time domain. More precisely, the analysis may be carried out using minimal and maximal values of time parameters similarly to the safety study of some railroad crossing in [9]. The other approach is probabilistic when time parameters are represented by probability distributions as in [2] where time coordination of distance protections in high voltage power transmission line was considered. The next kind of analysis will be based upon fuzzy set and will become the topic of the paper.

In this paper, the serious incident which occurred at the Chopin airport in Warsaw in 2007 year would be analyzed. Only static analysis will be executed, while dynamic one will be the topic of the following paper. In order to find the probability that given incident would transform into accident, the analysis of event trees by fuzzy probabilities will be performed.

2 Serious Air Traffic Incident No. 344/07

An analysis of incidents using fuzzy inference is illustrated with the example of a serious air traffic incident which occurred in August 2007 at the Warsaw Chopin airport between Boeing 767 and Boeing 737 aircraft. Its cause was classified as a "human factor" and the causal group H4 – "procedural errors" [18].

2.1 Description of the Circumstances of the Incident

In the incident on 13th of August 2007 participated two aircraft – Boeing 737 (B737) and the Boeing 767 (B767), which more or less at the same time were scheduled for take-off from the Warsaw Chopin airport. As the first, clearance for line-up and wait on runway 29 was issued to B737. As a second, clearance for line-up and wait on runway 33 was given to the B767 crew. The latter aircraft was the first to obtain permission to take-off. A moment after confirmation of permission to take-off, both aircrafts began the start procedure at the same time. The B737 crew wrongly assumed that the start permission was addressed to them. They probably thought that since they had received the permission to line up the runway first, they would be also the first to be permitted to start. In addition, the categories of wake turbulence caused that from the traffic efficiency point of view, it would be better to start B737 before B767. Decision of the controller, however, was different. The air traffic controller (ATC) did not watch the planes taking-off, because at this time he was busy agreeing a helicopter take-off. The situation of simultaneous start was, nevertheless, observed by the pilot of ATR 72, who was waiting in the queue for departure. He reacted over the radio. After this message, B767 pilot looked right and saw B737 taking-off. Then, on his own initiative, braked off and began a rapid deceleration, which led to stopping the plane 200 meters from the intersection of the runways. The assistant controller heard the ATR 72 pilot radio message and informed the controller that B737 operated without authorization. The controller, who originally did not hear the information on the radio, after 16 seconds from the start, recognized the situation and strongly ordered B737 to discontinue the take-off procedure. The B737 crew performed braking and stopped 200 m from the intersection of the runways.

2.2 Premises Conducive for Accident

In the presented example it can be noticed that it is sufficient to impose only one additional risk factor (or a combination of two factors), and the incident would become, in fact, an accident. There are several premises conducive for an accident [15].

1. Weather conditions (visibility) are so bad that it is impossible to see the actual traffic situation. This applies to B767, ATR 72 crews, and the air traffic controller.
2. ATR 72 pilot does not watch the situation on the runways, just waiting for permission to line-up the runway.
3. ATR 72 pilot observes the situation, but does not immediately inform about it on the radio, instead he discusses it with other members of his own crew.
4. B767 crew, busy with their own take-off procedure, does not pay attention to the message transmitted over the radio by the ATR 72 pilot.
5. B767 crew takes a wrong decision to continue the take-off, despite noting B737 aircraft. Such decision could arise, for example, with this reasoning: "there is no possibility to stop before the intersection, let B737 stop - after all, we have a permission to start, maybe we can pass the intersection before the B737", etc.
6. Assistant controller does not pay attention to the information given by radio by the ATR 72 pilot, or does not respond to it properly - does not inform the controller.
7. B737 crew does not react properly to the air traffic controller command and does not interrupt the take-off procedure.

2.3 Scenarios Leading to Accident

As indicated above, only a small number of conducive events is necessary for transformation of an incident into an accident. There are several scenarios that are considered in the context of this work. Logical dependencies between the scenarios leading to accidents and premises conducive for them, are schematically shown in Table 1. In this paper we assume designation of premises E_i, where $i \in \{1, \ldots, 7\}$ and e.g. E_1 =

"Insufficient visibility". We also have adopted following designations: 1 - a premise occurred, 0 - a premise did not occur, *n.r.* - the occurrence of premise is irrelevant to the transformation of an incident into an accident or there is only one reasonable premise value.

Table 1. Scenarios of transformation of the incident 344/07 into an accident

	1. Insufficient visibility (E_1)	2. ATR 72 does nt monitor (E_2)	3. ATR 72 does not warn (E_3)	4. B767 does not hear the warning (E_4)	5. B767 does not brake off (E_5)	6. Assistant does not inform (E_6)	7. B737 does not interrupt take-off (E_7)
Scenario 1	1	*n.r.*	*n.r.*	*n.r.*	*n.r.*	*n.r.*	*n.r.*
Scenario 2	0	1	*n.r.*	*n.r.*	*n.r.*	*n.r.*	*n.r.*
Scenario 3	0	0	1	*n.r.*	*n.r.*	*n.r.*	*n.r.*
Scenario 4	0	0	0	1	*n.r.*	1	*n.r.*
Scenario 5	0	0	0	1	*n.r.*	0	1
Scenario 6	0	0	0	0	1	1	*n.r.*
Scenario 7	0	0	0	0	1	0	1

The above scenarios of take-off continuation were determined using the event tree, whom analysis, for the sake of limited paper size, has been omitted.

2.4 Method of Incident Analysis

Estimating the probability of each scenario would allow to determine the probability of the accident occurring as a result of this incident. Unfortunately, realization of most of these scenarios, depends on immeasurable values not available for statistical analysis. For example, in scenario 2 it is impossible to determine, using measurement methods, how often staff focuses exclusively on their procedures and draws little attention to external events. The situation is similar in scenarios 5, 6 or 7, in which we have to deal with human error. Of course, such errors do happen, but it is difficult to estimate the statistical probability of them. We do not know the actual number of such errors (we know at most those errors which have consequences in air traffic events), nor we know the number of opportunities to commit them, so there is no reference necessary to estimate their frequency. In literature we can find some models for estimating the likelihood of operators (pilots and controllers) errors, with respect to the causes of aviation accidents. For example, [3] uses MIDAS human performance model together with a model for estimating the risk of accidents TOPAZ to analyze similar issues - probability of a collision at the junction of the runway and taxiway.

The above mentioned reasons are the basis for seeking probabilities of events conducive for accident in the area of expert assessments. These obviously are often ambiguous and imprecise, which makes us propose the use of fuzzy methods in the analysis of incidents. In this paper, we focus on finding expressions for the fuzzy probability of an accident, given that under these circumstances, the consequences would be catastrophic. For scenarios that lead to the continuation of take-off, formulas for the fuzzy likelihood of their realization will be presented, and the probability will be calculated. The basis will be the event tree analysis by fuzzy probability.

3 Probability Scale

In [13] the example of probability classification scheme was proposed. It is shown in Table 2 which contains both qualitative and quantitative definitions of likelihood categories of aircraft on-board system failure. A similar approach is presented in [5].

Table 2. Probability of occurrence definitions ([13])

	Extremely improbable	Very rare	Rare	Probable	Frequent
Qualitative definition	Should virtually never occur in the whole fleet life	Unlikely to occur when considering several systems of the same type, but nevertheless has to be considered as being possible	Unlikely to occur during the total operational life of each system but may occur several times when considering several systems of the same type	May occur once during total operational life of one system	May occur once or several times during operational life
Quantitative definition	$< 10^{-9}$ per flight hour	10^{-7} to 10^{-9} per flight hour	10^{-5} to 10^{-7} per flight hour	10^{-3} to 10^{-5} per flight hour	1 to 10^{-3} per flight hour

Values of both scales are not precise. Experts can interpret them in different manners. These values can be expressed using fuzzy set theory [9, 12]. Event tree analysis by fuzzy probability has been described in paper [7]. In this paper, fuzzy sets for fuzzy probabilities are expressed by discrete membership functions with a few real values. In our paper, membership functions of fuzzy sets for fuzzy probabilities are trapezoidal. Such functions are used in fault tree analysis by fuzzy probabilities in [16, 17].

Linguistic variable *Probability* is shown in Fig. 1, where it is illustrated in logarithmic scale. The variable has the following values: extremely improbable (*EI*), very rare (*VR*), rare (*RE*), probable (*PR*), frequent (*FR*). For values *VR*, *RE* i *PR*, trapezoidal functions with parameters (*a*,*b*,*c*,*d*) are as follows:

$$\mu_i(x;a,b,c,d) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a < x \le b \\ 1, & b < x \le c \\ \frac{d-x}{d-c}, & c < x \le d \\ 0, & x > d \end{cases} \tag{1}$$

where $i \in \{VR, RE, PR\}$

For values *EI* and *FR* , trapezoidal functions are the following:

$$\mu_{EI}(x;a,b,c,d) = \begin{cases} 0, & x < a = b \\ 1, & b \le x \le c \\ \frac{d-x}{d-c}, & c < x \le d \\ 0, & x > d \end{cases} \tag{2}$$

$$\mu_{FR}(x;a,b,c,d)=\begin{cases}0, & x\le a\\ \frac{x-a}{b-a}, & a<x\le b\\ 1, & <x\le c=d\\ 0, & x>d\end{cases} \tag{3}$$

In Table 2, probability scale for aircraft on-board system failure in flight is shown. These systems are very reliable. In the analyzed incident, unreliability concerns mainly human factor. In contemporary air traffic systems, human error frequency is much higher than aircraft on-board system failure frequency. Hence, new scale has been accepted with values of linguistic variable *Probability* given by parameters (*a*,*b*,*c*,*d*) as in Table 3, and illustrated in Fig. 1.

Table 3. Parameters of membership functions of linguistic variable *Probability* values

	a	*b*	*c*	*d*
μ_{EI}	10^{-9}	10^{-9}	10^{-8}	10^{-7}
μ_{VR}	10^{-8}	10^{-7}	10^{-6}	10^{-5}
μ_{RE}	10^{-6}	10^{-5}	10^{-4}	10^{-3}
μ_{PR}	10^{-4}	10^{-3}	10^{-2}	10^{-1}
μ_{FR}	10^{-2}	10^{-1}	1	1

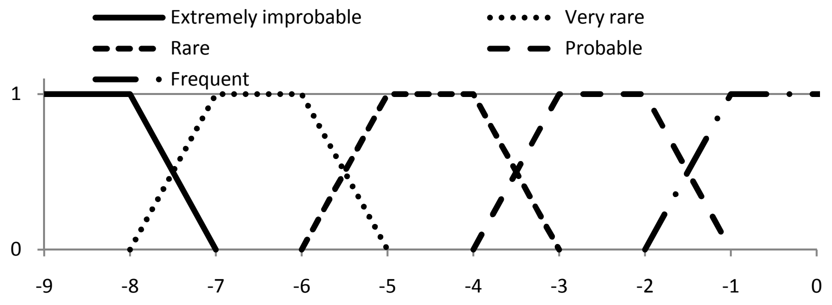

Fig. 1. Linguistic variable *Probability* in logarithmic form

4 Static Analysis of Scenarios Leading to Air Accident

We denote by P_1, P_2, ..., P_7 the probability of occurrence of premises conducive to formation of an aviation accident, and by $P(S_1)$, $P(S_2)$, ..., $P(S_7)$ - the probability of realization of scenarios leading to the transformation of the incident into accident. Fuzzy probabilities P_1, P_2, ..., P_7 will be determined on the basis of the literature, analysis of statistical data and expert assessments obtained for the present study. Such estimates are generally difficult to obtain and subject to a large margin of error, even if they are of fuzzy nature and therefore inherently imprecise. A broader discussion of the problems involved in the risk analysis of complex anthropotechnical systems, particularly in relation to air traffic, can be found in [4].

P_1 - the probability that the weather conditions are unfavorable and do not allow incident participants to notice hazards. Determining this probability will be based on the analysis of meteorological data for the Warsaw Chopin Airport in the last six years. Daily observations from the 33th week of the year and the 8th month of the year (including August 13) were considered together. The results of this analysis are shown in Table 4.

Table 4. Weather conditions for the Chopin Airport [19]

	maximum precipitation [mm/h]	minimum precipitation [mm/h]	mean precipitation [mm/day]	maximum visibility [km]	minimum visibility [km]	mean visibility [km]	weather events
8th month	36	0	2,07	30	6	13,7	rain, fog, thunderstorm
33th week	21	0	1,84	27,5	6	13,9	rain, fog, thunderstorm
13.08 2007	4	0	1,8	9,6	9,6	9,6	rain, fog, thunderstorm

Table 4 shows that in spite of fog and precipitation occurring during this period, visibility conditions are good enough that they do not interfere with observation of airfield. Therefore we assume fuzzy probability P_1 equal to "*very rare*". Of course, the analysis taking into account autumn, winter or night conditions will require the adoption of probability P_1 close to the opposite end of the scale.

P_2 - the probability that ATR 72 pilot does not observe the situation on runways. Under normal conditions, taxiing and preparing to take-off is very demanding and requires to focus on own tasks. There is no time for any observation of the environment. In the general case P_2 should be assumed equal to "*frequent*". But in this particular case, waiting in a queue for a take-off (especially lasting a long time) reduces the deficit of time and allows observation of the environment. In addition, the B737 was to take-off from the same runway as ATR 72, and preceded it on the taxiway, so the observation was natural and necessary activity. ATR 72 also heard the radio communication of all participants of the event. Considering the above, we assume the fuzzy probability P_2 to be equal "*probable*".

P_3 - describes the probability of the event, that the pilot who spotted the danger does not inform about it. As in that incident professionally trained pilots were involved, it must be assumed that the fuzzy probability P_3 is set to "*very rare*".

P_4 - the probability of the event, that the B767 crew does not pay attention or does not properly understand the message of danger. ATR 72 pilot's message was not clear - it only indicated the existence of an unusual situation. Somewhat similar probability was estimated in [14], where one of analyzed threats was an undetected warning of runway occupancy sensor. In this paper, we assume that the fuzzy probability P_4 value becomes "*probable*".

P_5 - describes the event of failure of emergency braking maneuver. Given the obviousness of this maneuver, but also proximity of the speed v_1, determining the boundary speed above which one should continue with the take-off, we assume that the fuzzy probability P_5 is equal to "*probable*" .

P_6 - determines the probability of no preventive action from ATC. Given that the controller was busy with other activities, but also the fact that his main task is to ensure the air traffic safety, the probability of failure to respond to the signals of danger P_6 must be considered within the scope of "*very rare*".

P_7 - the probability of refusal to execute the controller's command. Conscious refusal seems impossible. However, the B737 crew could not understand the instructions, or the speed v_1 could be exceeded, in which case an effective response is impossible. Given the above, we assume that the probability of fuzzy probability P_7 is set to "*rare*".

All fuzzy probabilities adopted for analysis are shown in Table 5.

Table 5. Fuzzy probability of premises conducive for accident

Premise	Fuzzy probability
E_1 - Insufficient visibility (P_1)	*very rare* (*VR*)
E_2 - ATR 72 does not monitor (P_2)	*probable* (*PR*)
E_3 - ATR 72 does not warn (P_3)	*very rare* (*VR*)
E_4 - B767 does not hear the warning (P_4)	*probable* (*PR*)
E_5 - B767 does not brake off (P_5)	*probable* (*PR*)
E_6 - Assistant controller does not inform (P_6)	*very rare* (*VR*)
E_7 - B737 does not interrupt take-off (P_7)	*rare* (*RE*)

Probabilities of realization of scenarios are as follows:

$$P(S_1) = P_1 \tag{4}$$

$$P(S_2) = (1 - P_1) \cdot P_2 \tag{5}$$

$$P(S_3) = (1 - P_1) \cdot (1 - P_2) \cdot P_3 \tag{6}$$

$$P(S_4) = (1 - P_1) \cdot (1 - P_2) \cdot (1 - P_3) \cdot P_4 \cdot P_6 \tag{7}$$

$$P(S_5) = (1 - P_1) \cdot (1 - P_2) \cdot (1 - P_3) \cdot P_4 \cdot (1 - P_6) \cdot P_7 \tag{8}$$

$$P(S_6) = (1 - P_1) \cdot (1 - P_2) \cdot (1 - P_3) \cdot (1 - P_4) \cdot P_5 \cdot P_6 \tag{9}$$

$$P(S_7) = (1 - P_1) \cdot (1 - P_2) \cdot (1 - P_3) \cdot (1 - P_4) \cdot P_5 \cdot (1 - P_6) \cdot P_7 \tag{10}$$

Let us denote by K the event that both aircraft will continue the take-off, and by $P(K)$ the probability of that event, which is given by the expression:

$$P(K) = P_1 + (1 - P_1) \cdot \Big(P_2 + (1 - P_2) \cdot \Big(P_3 + (1 - P_3) \cdot \big(P_4 \cdot (P_6 + (1 - P_6) \cdot P_7) + (1 - P_4) \cdot P_5 \cdot (P_6 + (1 - P_6) \cdot P_7)\big)\Big)\Big) \tag{11}$$

Let us consider two trapezoidal fuzzy numbers $P_i = (a_i, b_i, c_i, d_i)$ and $P_j = (a_j, b_j, c_j, d_j)$. Their addition, subtraction, and multiplication, respectively, are represented by trapezoidal fuzzy numbers $(a_i + a_j, b_i + b_j, c_i + c_j, d_i + d_j)$, $(a_i - d_j, b_i - c_j, c_i - b_j, d_i - a_j)$, $(a_i \cdot a_j, b_i \cdot b_j, c_i \cdot c_j, d_i \cdot d_j)$ [16, 17].

Fuzzy probability of realization of scenarios $P(S_1)$, ..., $P(S_7)$ and fuzzy probability that both aircraft will continue the take-off is given in table 6.

Table 6. Fuzzy probabilities of scenarios realization

	a	*b*	*c*	*d*
$P(S_1)$	10^{-8}	10^{-7}	10^{-6}	10^{-5}
$P(S_2)$	$9{,}9999 \cdot 10^{-5}$	10^{-3}	10^{-2}	10^{-1}
$P(S_3)$	$8{,}9999 \cdot 10^{-9}$	$9{,}9 \cdot 10^{-8}$	$9{,}99 \cdot 10^{-7}$	$9{,}999 \cdot 10^{-6}$
$P(S_4)$	$8{,}9998 \cdot 10^{-13}$	$9{,}9 \cdot 10^{-11}$	$9{,}99 \cdot 10^{-9}$	$9{,}999 \cdot 10^{-7}$
$P(S_5)$	$8{,}9997 \cdot 10^{-11}$	$9{,}9 \cdot 10^{-9}$	$9{,}99 \cdot 10^{-7}$	$9{,}999 \cdot 10^{-5}$
$P(S_6)$	$8{,}0998 \cdot 10^{-13}$	$9{,}801 \cdot 10^{-11}$	$9{,}98 \cdot 10^{-9}$	$9{,}998 \cdot 10^{-7}$
$P(S_7)$	$8{,}0998 \cdot 10^{-11}$	$9{,}801 \cdot 10^{-9}$	$9{,}98 \cdot 10^{-7}$	$9{,}998 \cdot 10^{-5}$
$P(K)$	$1{,}0002 \cdot 10^{-4}$	$1{,}0002 \cdot 10^{-3}$	$1{,}0004 \cdot 10^{-2}$	$1{,}0022 \cdot 10^{-1}$

In order to calculate the value of linguistic variable *Probability* for fuzzy probability $P(K)$, one can apply Jacard's similarity of two fuzzy sets. Jacard's similarity of fuzzy sets A, B with membership functions μ_A, μ_B is defined by [11]:

$$s_J(A,B) = \frac{\int_X min\big(\mu_A(x), \mu_B(x)\big)\, dx}{\int_X max\big(\mu_A(x), \mu_B(x)\big)\, dx} \tag{12}$$

As we adopted logarithmic scale for linguistic variable *Probability*, the formula for Jacard's similarity calculations was modified to the following form:

$$s_J^{\log}(A,B) = \frac{\int_X min\big(\mu_A(x), \mu_B(x)\big) \log_{10} dx}{\int_X max\big(\mu_A(x), \mu_B(x)\big) \log_{10} dx} \tag{13}$$

For each value of linguistic variable *Probability EI*, *VR*, etc., similarity with trapezoidal fuzzy number $P(K)$ was calculated and is given in Table 7.

Table 7. Jacard's similarity calculation results

Value of linguistic variable *Probability* (*ProbVal*)	$s_J^{\log}(P(K), ProbVal)$
extremely improbable (*EI*)	0
very rare (*VR*)	0
rare (*RE*)	$6{,}6646 \cdot 10^{-2}$
probable (*PR*)	$9{,}9967 \cdot 10^{-1}$
frequent (*FR*)	$7{,}7038 \cdot 10^{-2}$

In the analyzed air traffic incident none of the premises E_i did actually occur. However, there is no certainty that this is a permanent property. Institutions responsible for the air traffic safety take many preventive actions to eliminate the factors that contribute to accidents and incidents. The important question is, which factors should be eliminated first and which merit the most attention. For each premise E_i, we want to find fuzzy probability $P(K|\neg E_i)$ that both aircraft will continue the take-off provided this premise is not true. These fuzzy probabilities will allow evaluation of consequences of preventive activities.

$$P(K|\neg E_1) = P_2 + (1 - P_2) \cdot \Big(P_3 + (1 - P_3) \cdot \big(P_4 \cdot (P_6 + (1 - P_6) \cdot P_7) + (1 - P_4) \cdot P_5 \cdot (P_6 + (1 - P_6) \cdot P_7)\big)\Big) \tag{14}$$

$$P(K|\neg E_2) = P_1 + (1 - P_1) \cdot \Big(P_3 + (1 - P_3) \cdot \big(P_4 \cdot (P_6 + (1 - P_6) \cdot P_7) + (1 - P_4) \cdot P_5 \cdot (P_6 + (1 - P_6) \cdot P_7)\big)\Big) \tag{15}$$

$$P(K|\neg E_3) = P_1 + (1 - P_1) \cdot \Big(P_2 + (1 - P_2) \cdot \big(P_4 \cdot (P_6 + (1 - P_6) \cdot P_7) + (1 - P_4) \cdot P_5 \cdot (P_6 + (1 - P_6) \cdot P_7)\big)\Big) \tag{16}$$

$$P(K|\neg E_4) = P_1 + (1 - P_1) \cdot \Big(P_2 + (1 - P_2) \cdot \big(P_3 + (1 - P_3) \cdot P_5 \cdot (P_6 + (1 - P_6) \cdot P_7)\big)\Big) \tag{17}$$

$$P(K|\neg E_5) = P_1 + (1 - P_1) \cdot \Big(P_2 + (1 - P_2) \cdot \big(P_3 + (1 - P_3) \cdot P_4 \cdot (P_6 + (1 - P_6) \cdot P_7)\big)\Big) \tag{18}$$

$$P(K|\neg E_6) = P_1 + (1 - P_1) \cdot \Big(P_2 + (1 - P_2) \cdot \big(P_3 + (1 - P_3) \cdot (P_4 \cdot P_7 + (1 - P_4) \cdot P_5 \cdot P_7)\big)\Big) \tag{19}$$

$$P(K|\neg E_7) = P_1 + (1 - P_1) \cdot \Big(P_2 + (1 - P_2) \cdot \big(P_3 + (1 - P_3) \cdot (P_4 \cdot P_6 + (1 - P_4) \cdot P_5 \cdot P_6)\big)\Big) \tag{20}$$

Jacard's similarity between $P(K|\neg E_i)$, where $i \in \{1, \dots, 7\}$, and values of linguistic variable *Probability* is given in Table 8.

Table 8. Jacard's similarity between $P(K|\neg E_i)$, where $i \in \{1, \dots, 7\}$, and values of linguistic variable *Probability*

	$P(K\mid\neg E_i)$				$s_J^{\log}(P(K\mid\neg E_i), ProbVal)$				
	a	b	c	d	EI	VR	RE	PR	FR
$i=1$	$1{,}0001 \cdot 10^{-4}$	$1{,}0001 \cdot 10^{-3}$	$1{,}0003 \cdot 10^{-2}$	$1{,}0021 \cdot 10^{-1}$	0	0	$6{,}6652 \cdot 10^{-2}$	$9{,}9971 \cdot 10^{-1}$	$7{,}7031 \cdot 10^{-2}$
$i=2$	$2{,}0192 \cdot 10^{-8}$	$2{,}201 \cdot 10^{-7}$	$4{,}019 \cdot 10^{-6}$	$2{,}2199 \cdot 10^{-4}$	$2{,}9378 \cdot 10^{-2}$	$5{,}6335 \cdot 10^{-1}$	$2{,}7520 \cdot 10^{-1}$	$4{,}7242 \cdot 10^{-3}$	0
$i=3$	$1{,}0001 \cdot 10^{-4}$	$1{,}0001 \cdot 10^{-3}$	$1{,}0003 \cdot 10^{-2}$	$1{,}0021 \cdot 10^{-1}$	0	0	$6{,}6652 \cdot 10^{-2}$	$9{,}9971 \cdot 10^{-1}$	$7{,}7031 \cdot 10^{-2}$
$i=4$	$1{,}0002 \cdot 10^{-4}$	$1{,}0002 \cdot 10^{-3}$	$1{,}0003 \cdot 10^{-2}$	$1{,}0012 \cdot 10^{-1}$	0	0	$6{,}6651 \cdot 10^{-2}$	$9{,}9979 \cdot 10^{-1}$	$7{,}6988 \cdot 10^{-2}$
$i=5$	$1{,}0002 \cdot 10^{-4}$	$1{,}0002 \cdot 10^{-3}$	$1{,}0003 \cdot 10^{-2}$	$1{,}0012 \cdot 10^{-1}$	0	0	$6{,}6651 \cdot 10^{-2}$	$9{,}9979 \cdot 10^{-1}$	$7{,}6988 \cdot 10^{-2}$
$i=6$	$1{,}0002 \cdot 10^{-4}$	$1{,}0002 \cdot 10^{-3}$	$1{,}0004 \cdot 10^{-2}$	$1{,}0022 \cdot 10^{-1}$	0	0	$6{,}6646 \cdot 10^{-2}$	$9{,}9967 \cdot 10^{-1}$	$7{,}7037 \cdot 10^{-2}$
$i=7$	$1{,}0002 \cdot 10^{-4}$	$1{,}0002 \cdot 10^{-3}$	$1{,}0002 \cdot 10^{-2}$	$1{,}0002 \cdot 10^{-1}$	0	0	$6{,}6655 \cdot 10^{-2}$	$9{,}9991 \cdot 10^{-1}$	$7{,}3938 \cdot 10^{-2}$

Calculations for the basic variant (Tables 6 and 7) show that the fuzzy likelihood of take-off continuation is most compliant with a value "*probable*" (*PR*) of linguistic variable *Probability*.

Analysis of the results of calculations in Table 8 shows that elimination of premises E_1, E_3, E_4, E_5, E_6 and E_7 does not change the above mentioned fuzzy evaluation of the possibility of transformation from incident into accident. The most important in this case is the premise E_2 - "ATR72 does not monitor". It turns out that preventive action aiming at the elimination of this premise moves the evaluation of linguistic variable *Probability* into the area between "*rare*" (*RE*) and "*very rare*" (*VR*) values. This means a significant increase in the level of safety. Elimination (or reduction of the likelihood) of the premise E_2 is practically possible. Pilot training should be carried out to increase the understanding of the need to monitor the airfield during the taxiing procedure and while waiting for permission to take-off. One can also consider the introduction of recommendation to carefully observe other traffic into the operating instructions.

5 Summary

In the paper serious incident which occurred at the Chopin airport in Warsaw in 2007 year has been analyzed. While the static analysis has been carried out, the dynamic one will be treated in the next paper. Probability scale for events has been given and it is five values one. For the values the suitable fuzzy sets have been defined. Scenarios of transformation of the incident into an accident have been found using event tree. Fuzzy probability of the transformation has been calculated. Finally, it has been shown that elimination of one of premises for transformation of the incident into accident significantly reduces the possibility of this transformation.

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