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Analysis of Air Traffic Incidents using event trees with fuzzy probabilities

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Abstract

This paper presents a methodology for analysing Air Traffic Incidents based on an estimate of the probability of an Air Traffic Incident developing into an Air Traffic Accident (Air Traffic Incidents are officially defined as events which are dangerous but without a catastrophic impact). A new approach is presented as previous research in this area has focused more on defining accident occurrence probabilities. Access to detailed information on Air Traffic Incidents is limited and for this reason the methodology developed was based on Fuzzy Set Theory. Analysis was carried out using Event Trees leading from a real Incident to a hypothetical Accident with the probabilities of occurrence of the various scenarios being defined by fuzzy sets. The results of this analysis enable the calculation of the fuzzy probability of the Incident being transformed into an Accident. Performing this analysis, new measuring techniques for the comparison of fuzzy sets were developed and partially verified. The Case Study presented in the paper analyses a Serious Runway Incursion Incident. This Serious Incident is analysed for influencing factors such as: pilot and flight controller skill levels, airport traffic volume, weather conditions, airport procedures and airport geometry. Applying the methods presented in this paper enables an assessment of the effectiveness of Preventive Recommendations of Accident/Incident Investigation Commissions. Additionally, probability estimations can be developed for different incident situations allowing the identification of Security System Weak Points thus enabling appropriate proactive measures to be taken for their elimination.

Keywords: Air traffic safety; Event tree; Fuzzy probability; Linguistic modelling; Serious incident; Transportation

1. Introduction

Air transport is generally thought as the safest transport mode as Passenger Safety is the business' top priority. Technical, organisational and procedural methods are applied to reduce as much as possible the risk of occurrence of air traffic accidents. Sometimes, these methods fail, in most cases because of human error, and the resulting accidents

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are investigated to identify root causes and help define preventive measures. The results of these investigations are usually qualitative [31].

Over the last few years, air transport development has resulted in attempts to standardise risk management tools and methodologies with a specific focus on defining acceptable safety levels. As a result, in 2001, the European Organisation for the Safety of Air Navigation (Eurocontrol) issued six documents relating to safety standards (ESARR requirements – Eurocontrol [12]). The minimum safety requirements used by European Aviation Authorities are those set by the ECAC (European Civil Aviation Conference) which were adapted by Eurocontrol in the ESARR-4 Regulations. These regulations divide events which involve the participation of ATM (Air Traffic Management) into 5 categories: Accidents, Serious Incidents, Major Incidents, Significant Incidents, Incidents (i.e. incidents with no immediate safety impact). A "Safety Tolerance" is defined only for the "Accidents" category where an indicator called TLS (Target Level of Safety) sets the maximum probability of an accident involving commercial aircraft at $1.55 \cdot 10^{-8}$ accidents per flight hour and at $2.31 \cdot 10^{-8}$ accidents per flight [12].

All ECAC Member States are obliged to assess their CLS (Current Level of Safety) and compare it to the TLS. They are also required to make a forecast of changes in Safety Levels in future years and to propose suitable remedies should the forecasts exceed the accepted standard. This task is complicated by the fact that TLS is based on number of accidents with regard to volume of traffic making it impossible to provide a reliable determination of CLS in the many countries where there have been no air accidents in recent years.

Air traffic incident investigation is regulated by the following National and International Regulations: Annex 13 to the Chicago Convention [19] and EU Regulation 996/2010 on the Investigation and Prevention of Accidents in Civil Aviation [15]. Both regulations define three categories of events:

- Accident – An event associated with the operation of the aircraft, which occurred whilst people were on board, during which any of the persons suffered serious injury or during which the aircraft was damaged.
- Serious Incident – An incident where an accident almost took place and was caused by, for instance: significant violation of aircraft separation requirements where the situation was controlled neither by the Aircraft Pilot nor by the Flight Controller.
- Incident – An event associated with the operation of an aircraft which would adversely affect safety but where the situation was under the control either of the Aircraft Pilot or of the Flight Controller (e.g. a violation of the separation between aircraft but with the Pilot or the Flight Controller controlling of the situation).

Investigation of Air Traffic Accidents has four aims: identification of root causes, establishment of the circumstances contributing to event occurrence, recommendations regarding preventive measures for root cause elimination and steps required in order to prevent analogous situations in future.

There are numerous techniques of a technical, procedural, organisational or management nature which are designed to prevent accidents. However, the occurrence of accidents indicates these techniques sometimes fail. Therefore, when an accident occurs, the situation is analysed, root causes are identified and technical, procedural, organisational or management structures are modified. This activity is reactive. It would not take place if there was no accident and it is in this specific context that this paper seeks to define a proactive approach on the subject.

The research problem discussed in the paper is an attempt to provide quantitative answers to questions regarding the degree to which a Serious Air Traffic Incident could result in an Air Traffic Accident. Quantitative answers are needed because they enable identification of the "weakest links" in the "chain" of technical, procedural, organisational or management techniques that could lead to an event of this nature. From this perspective, the research problem can be described as follows:

1. Estimation of the probability of an Air Traffic Incident becoming an Air Traffic Accident.
2. Analysis of the operational efficiency of the technical, procedural, organisational and management structure in the context of the Air Traffic Incident.

1.1. Literature overview

One method for solving the lack of availability of data for CLS assessment is to use data on Air Incidents which are (obviously) more frequent than Air Accidents. The validity of this approach is recognised because the Serious Incidents Rate is a key safety related indicator within the aviation industry [18]. This approach proposes that if the CLS value based on incidents is within the TLS limits specified for accidents, it can be assumed that this is a satisfactory result not requiring further research [7]. However, a more detailed analysis of serious incidents in transport can be found in [48]. This analysis shows that the reasons a serious incident does not necessarily develop into an accident are sometimes as "mundane" as (for instance) the occurrence of heavy traffic. Standard analyses of accident causes and resulting preventive measure recommendations do not take these issues into account and [56] noted that during accident analysis it may be difficult to determine the logical relationships between all the events connected with an accident. For this reason, an Incident Tree Analysis Methodology is proposed that characterises information flow as being within a system rather than as logical relationships. Moreover, in their paper [56], use a different definition of incident than that applied in this paper. For [56], poor weather conditions, improper perception action and steering failure are regarded as incidents. As a result, in the Incident Tree an accident is assigned to the Tree Root whilst incidents are assigned to the other vertices. In spite of these differences, the accident formation process model presented by Wang et al. [56], can be applied to study serious air traffic incidents. Relationship and Lack of Relationship between incidents is represented by arcs with binary values of 1 and 0 respectively. Accident formation analysis is based on more precise available information than in the study discussed in this paper. Moreover, based on an example of an accident which took place as a vehicle was leaving a roadway, a possible approach is presented to identify the: uncertain, random, complex, possible and variable characteristics of accident occurrence. Jacquemart and Morio [26], propose a concept called dynamic importance which is based on splitting algorithms into an incident or an accident probability estimation. The main idea is to arrive at an estimate of several conditional probabilities that are easier to estimate through a (very tough) simulation rather than by attempting to estimate their conflict probability. Ma et al. [35] propose analysing the catastrophic accident as a fuzzy event, and then to identify the most possible catastrophic accident sequence. They propose a risk evaluation system consisting of a credibility measure, a global fuzzy severity measure and a risk measure based on credibility theory. If human error is a part of an analysed incident (as in the case of the incident presented in this paper) then according to [8] a combination of Fault Tree Analysis and Task Analysis is useful.

In safety and reliability as well as in risk analysis and management, information is often uncertain and imprecise thus, in their book [37] present three methods for estimating safety and reliability when this is based on uncertain and imprecise information:

– Probability and statistics,
– Fuzzy set theory,
– Possibility theory (inspired by the above).

In a later paper, [1] present the following approaches for the representation of uncertainty:

– Probability,
– Imprecise (interval) probability,
– Probability bound analysis,
– Possibility theory (foundations: probability, statistics, fuzzy sets)
– Dempster–Shafer evidence theory.

The most popular tools that are applied in safety and reliability as well as in risk analysis and management are Fault Trees and Event Trees. Fault Trees are system structure oriented, whereas Event Trees are scenario oriented. The goal of Fault Tree Analysis is to find the probability characteristic of a system top event (hazard, accident) based on the probabilistic parameters of system component failures. Events are binary (they occur or they do not occur) and are statistically independent. Relationships between events are expressed by logical (binary) gates. In Event Trees, the initialising event is associated with the leftmost edge of the tree (see Fig. 1). Each level is assigned to an event and at each level branching is determined by the following cases: either the event occurs or the event does not occur. The sequence of cases from a root to a leaf represent a scenario enabling a probability calculation of scenario occurrence. Fault Trees with fuzzy probabilities are presented in [51,53,40,41] while Event Trees with fuzzy probabilities are described in [30]. Fault trees with probabilities expressed by intervals are studied in [25]. In [3], uncertainty in event tree analysis is studied using Monte Carlo simulation and possibility theory. When analysing incidents, time can be accommodated either as a qualitative factor (i.e. as a sequence of events over time) or as a quantitative factor. Accommodating time as a quantitative factor enables execution of a timing analysis which can be performed using minimum and maximum values for time parameters. An example of this is a safety study of a railroad crossing described in [36]. Other approaches to accommodating time are probabilistic ones where time parameters are represented by probability distributions. An example of this approach in [2] presents time coordination of distance protection in high voltage power transmission lines. Other methods of analysis are based on fuzzy sets.

Bayesian Networks [27] are also used in safety and reliability, risk analysis and management [33,52]. Influence Diagrams [22,45] are Bayesian Networks enhanced by Action (i.e. Decision) Nodes, Utility (i.e. Value) Nodes and appropriate arcs.

A number of papers [28,58], describe transformation of Event Trees into Influence Diagrams. Possibilities of Branching are normally uncertain therefore only rarely can they be expressed by a point value. For instance, in the transformation presented in [28], branching probabilities are characterised by beta distribution and posterior beta distribution is estimated using a combination of anterior beta distribution and observation. In the study presented in this paper, a number of the scenario occurrences include observed data. There are similarities between Event Trees with fuzzy probabilities and Influence Diagrams in that branching probabilities using these techniques are characterised by an interval and not by a point value. Moreover in both, branching probabilities can be assessed by integration of experts' judgements. An example of this is in a paper by Jun et al. [28], where the anterior beta distribution was calculated using experts' judgements. However, according to [58], the most significant disadvantage of Event Tree based models compared to Influence Diagrams is that there is no mechanism for adaptively modifying model parameters. This would affect (for instance) branching probabilities in that they cannot be modified as a result of observed data. However, when analysing a Serious Air Traffic Incident it is often impossible to gather data on the numbers of scenario occurrences and therefore, this advantage of Influence Diagrams is somewhat less relevant in Serious Incident Analysis.

When experts carrying out multi-criteria assessments do not agree, the approach presented by [49] may be useful. Multiple criteria group evaluation of alternatives under uncertainty is applied to an incident where there was a problem in identifying the most important causes.

In another paper [39] group decision making problems were applied to the Risk Assessment Problem. This approach was based on using an ordered weighted average aggregation operator and fuzzy preference relations for any pair of alternatives.

1.2. Proposed approach

The approach proposed in this paper is to estimate the fuzzy probability of the incident described transforming into an accident based on accident formation scenarios. A limitation of this is that realisation of some scenarios depends on values which "by their nature" cannot provide a basis for statistical analysis. One example of this is the impossibility of measuring how often members of staff work on tasks in compliance with procedures as opposed to spending this time monitoring external events. Another example is human error. It is impossible to estimate the statistical probability of human error because there is no record of the actual number of these errors. All that exists as information on human errors is whatever is reported in external analyses such as air traffic events. This absence of data is compounded by the fact that, for obvious reasons, there is no record of the opportunities when human error could have been "committed" irrespective of the circumstances. For these reasons, neither is there any record of actual human errors committed by operators (pilot or controller), nor is there any human error reference point against which the probability of human error could be estimated.

Literature on the causes of aviation accidents presents some models which could be used for estimating the likelihood of operator error. For example, Blom et al. [4] use a combination of the MIDAS human performance model, and the TOPAZ accident risk model to analyse similar issues (for instance an analysis of collision probability at runway and taxiway junctions).

For the statistical difficulties described above experts' assessments are normally used to provide a basis for defining the probabilities of events taking place which could cause accidents. However, these experts' assessments are often ambiguous and imprecise and for these reasons, this paper presents the use of fuzzy methods for incident analysis. The paper focuses on finding appropriate expressions for the fuzzy probability of an accident of catastrophic consequences.

Using Event Tree Analysis with fuzzy probability, formulae will be presented for fuzzy probabilities for scenarios leading to an accident.

The proposed approach is as follows:

1. Computation of the probability of transformation of a Serious Incident into an Accident using an Event Tree with fuzzy probabilities,
2. If the computed probability is too great (i.e. Probability of Flight Accident is greater than $2.31 \cdot 10^{-8}$ [12], then the approach is to execute: a) an importance analysis applying hypothetical prevention of events in order to identify the means (safety barriers) necessary to improve safety, b) a sensitivity analysis. Both are executed using Event Trees with fuzzy probabilities,
3. If the means identified in point 2 are not sufficient an analysis of other safety oriented facilities is performed.

The proposed approach will be validated on a Serious Incident which occurred at Chopin airport in Warsaw in 2007. This case belongs to a class defined as: Runway Incursions (RI's). The International Civil Aviation Organization (ICAO) defines RI's as "any occurrence at an aerodrome involving the incorrect presence of an aircraft vehicle or person on the protected area of a surface designated for the landing and take-off of aircraft" [21]. The most common RI sub-classes are:

– Take-off without air traffic controller (ATC) permission (also includes take-off from wrong runways, from taxiways or from outside the runway thresholds).
– Crossing the aircraft runway after landing contrary to ATC clearance.
– ATC taxi clearance issued in conflict with another ATC clearance.
– Unauthorised runway incursion of people, vehicles or animals.

The serious incident analysed in this paper belongs to the "take-off without the ATC permission" sub-class. There are many factors that affect this type of event: weather, airport configuration, conditional control clearances, simultaneous use of intersecting runways, phraseology, use of several languages for controller-pilot communication, workload and several more. It is a very broad sub-class with extremely serious possible consequences. For this reason a lot of attention is devoted to it in the activities of organisations responsible for air traffic safety. According to Eurocontrol statistics for the period 2008–2011 the number of RI's was close to 0.06 per 10,000 flights [14]. According to the U.S. FAA (Federal Aviation Administration) [16], in the airspace of the United States from 2004 to 2007, 1353 RI's were recorded for about 248 million aircraft take-offs and landings giving $5.46 \cdot 10^{-6}$ per flight (i.e. a similar statistic to that in Europe).

The current strategy of the European Aviation Safety Agency (EASA) is to focus on five key operational problems in air traffic – the "Top Five". This list also includes RI's.

The Incident analysed is "typical" for the RI sub-class. Contributory factors are also typical such as: simultaneous use of intersecting runways, lack of situational awareness as a result of a conditional clearance, heavy workload of crew. Comparing this approach to other research for instance [18], the results of the approach presented in this paper allow an assessment of the likelihood of an accident at an organisational, human or technical level. Both approaches can be applied to assess the impact of organisational changes. However, the approach presented in this paper is based on an analysis of the logical dependence between events rather than on an analysis of statistical data relating to real accidents. The approach presented can therefore be used to develop proactive safety indicators (which is an area strongly emphasised in this paper). Compared to [48], the approach and analysis presented in this paper should make it easier to take into account subjective factors specifically, those related to the human factor. Wang et al. [56] uses a concept of the "amount of information of the fuzzy incident" in a case study dealing with road traffic. Both Incident Tree Analysis and the approach presented in this paper allow an assessment of the likelihood of an accident formation. In this paper however, the authors identify changes of probability that apply in conditions which are different from those in a real incident. In comparison with [35], whilst both papers present a fuzzy approach to analysis of an accident with catastrophic consequences, in this paper the analysis begins with a situation that is classed as a Serious Incident. This approach greatly reduces analysis scope by permitting more detailed analysis of the factors which lead to an accident as the last phase of the development of the event. This approach enables the definition of actions which being proactive, can be much more effective. Comparison with the work of [8], which presents no numerical results, the analysis presented in this paper allows a quantitative consideration of the fuzzy probability of human error.

This paper is an extended version of a previous paper prepared by the authors [34]. In this previous paper a logarithmic Jaccard similarity was used in order to determine the value of the linguistic variable *Probability*. In the present paper, a logarithmic subsethood measure is used. In the present paper it will be proved that the sum of logarithmic Jaccard's similarities over all linguistic variable *Probability* values for a fuzzy set can be much smaller than 1. However, it will be proved that the sum of logarithmic subsethood measure over all linguistic variable *Probability* values is in the interval $[1, 2]$. It is reason why the second measure is used. Additionally, in this present paper, the previous paper [34] is extended in the following areas: definition of logarithmic subsethood measure, properties of logarithmic Jaccard similarity and logarithmic subsethood measures, sensitivity analysis, case analysis of other conditions than those that are directly connected with the incident, analysis of accident prevention recommendations issued by the State Commission for Aircraft Accident Investigation.

This paper is structured in 7 sections. Section 1 consists of the introduction, literature review and presentation of proposed approach. Section 2 is a description of a serious Air Traffic Incident that is the case study used in this paper. Section 3 is a presentation of the fuzzy probability scale plus a new measure for determining the values of this scale. Properties of this measure are presented too. Section 4 (divided into three sub-sections) presents the calculation of the fuzzy probabilities for scenarios potentially leading to an Air Traffic Accident. Section 4.1 presents the probability of transformation of the Serious Incident into an Accident. Section 4.2 presents an analysis of the importance of the events connected with the Serious Incident. Section 4.3 presents a sensitivity analysis of these events. Study in Section 4 is for conditions that occurred in the Incident. Calculations of the probabilities of transformation of the Serious Incident into an Accident for conditions different than those actually occurring are presented in Section 5. Section 6 consists of an analysis of the accident prevention recommendation issued by the State Commission for Aircraft Accident Investigation. This analysis is performed for the conditions that occurred in the Serious Incident and for conditions that are different from those in the Serious Incident. The final section is a presentation of the results with an appropriate discussion.

2. Serious Air Traffic Incident No. 344/07

The case study chosen for analysis of incidents using fuzzy inference is a Serious Air Traffic Incident which occurred in August 2007 at Warsaw Chopin airport between a Boeing 767 and a Boeing 737 aircraft. Per a Government Report (Urząd Lotnictwa Cywilnego [54]), the root cause was classified as “Human Factor” under Causal Group H4 – “Procedural Errors”.

2.1. Description of the circumstances of the incident

On 13th Aug. 2007 two aircraft, a Boeing 767 (B767) and a Boeing 737 (B737) were scheduled for take-off more or less at the same time from Warsaw Chopin airport. Clearance for runway line-up and wait (runway 29) was issued first to the B737. Clearance for runway line-up and wait (runway 33) was then given to the B767. The B767 was therefore the second aircraft to be given clearance for runway line-up and wait. However, the B767 was the first aircraft to obtain permission for take-off. A moment after confirmation of take-off permission, both aircraft began the start procedure at the same time. The B737 crew had assumed that the start permission was addressed to their aircraft probably thinking that as his aircraft was the first to receive permission for runway line up, it would also be the first to be given take-off permission. Additionally, wake turbulence categories indicated that, from a traffic efficiency point of view, it would be better to start the B737 before the B767. However, the decision of the controller was different. The air traffic controller (ATC) did not watch planes' take-off, because at this time he was busy agreeing a helicopter take-off. This situation where both aircraft were preparing to take-off at the same time was noticed by the pilot of an ATR72 aircraft, which was standing in-queue waiting for departure. He reacted on the radio. Hearing this message, the B767 pilot looked right and saw the B737 taking-off. Then, on his own initiative, the B767 pilot broke off and began a rapid deceleration, which led to the plane stopping 200 meters from the intersection of both runways. The assistant air traffic controller heard the ATR72 pilot's radio message and informed the controller that the B737 had operated without authorisation. The controller, who did not hear the radio message from the ATR72 pilot, noted the situation and 16 seconds from the start, strongly ordered the B737 to discontinue the take-off procedure. The B737 crew braked their aircraft and stopped 200 m from the intersection of the runways.

2.2. Premises conducive for Accident

In the case study presented it can be noticed that it is sufficient to impose only one additional risk factor (or a combination of two factors), and the incident would have been an Accident because there are a number of factors that could be conducive events causing accident occurrence [47]:

1. Weather conditions (visibility) making it impossible to see the actual traffic situation. This is a risk factor which could apply to the crew of the B767, the crew of the ATR72 and the air traffic controller.
2. The ATR72 crew were not observing the runway situation but waiting for permission for runway line-up.
3. The ATR72 pilot observes the situation, but does not immediately take radio action because for instance, there is a discussion with other ATR72 crew members.
4. The B767 crew, busy with their own take-off procedure, do not pay attention to the message transmitted by the ATR72 pilot.
5. The B767 crew take a decision to continue take-off, despite observing the B737 aircraft. Such decisions could arise, for example by applying reasoning such as: "there is no possibility to stop before the intersection so let the B737 stop – after all, we have a permission to start and this way maybe we can cross the intersection before the B737" etc.
6. The assistant controller does not pay attention to the information given by the ATR72 pilot or does not respond to it in an appropriate manner and does not (for instance) inform the controller.
7. The B737 crew does not react properly to the air traffic controller's command and does not interrupt take-off procedures.

2.3. Scenarios leading to Accident

As indicated above, only a few conducive events are necessary for the incident to be transformed into an accident. In this context, a number of scenarios can be considered which are determined according to the Event Tree presented in Fig. 1. For Scenario 1 (S1), if event "Insufficient visibility" occurs, then it is represented by symbol T (true). For Scenario 2, if visibility is sufficient, then it is expressed by symbol F (false). If event "ATR72 does not monitor" occurs, then it is represented by T etc.

Logical dependencies between scenarios leading to an accident and premises conducive for them, are schematically shown in Table 1. In this paper, designation of premises (events) is indicated by: E_i, where $i \in \{1, \dots, 8\}$. For example, E_1 = "Insufficient visibility". Designations are presented as follows: 1 – premise occurred, 0 – premise did not occur, n.r. – either case applies where the occurrence of a premise is irrelevant to the transformation of an incident into an accident or, there is only one reasonable premise value.

3. Probability scale

3.1. Membership functions of values of linguistic variable probability

ICAO [20] proposes a probability scale scheme containing definitions of likelihood categories for aircraft on-board system failure during flight (shown in Table 2). A similar approach is presented in [6].

Scale classification values are not precise and experts interpret them in different ways. However, these values can be expressed using fuzzy set theory [29,44]. Event Tree analysis by fuzzy probability has been described in a paper by [30] and in his paper, fuzzy sets for fuzzy probabilities were expressed by discrete membership functions with few real values. In the present paper, membership functions of fuzzy sets for fuzzy probabilities are trapezoidal. Such functions are used in Fault Tree analysis by fuzzy probabilities in [51,53]. The general motivation to use fuzzy approach to the probability scale was the lack of consensus to the probability scale among aviation experts. We may observe this for instance in official ICAO documents. Safety Management Manual (Doc 9859) [20] expresses the probability with the use of numerical interval scale only in the first edition. In the following editions, only linguistic terms are

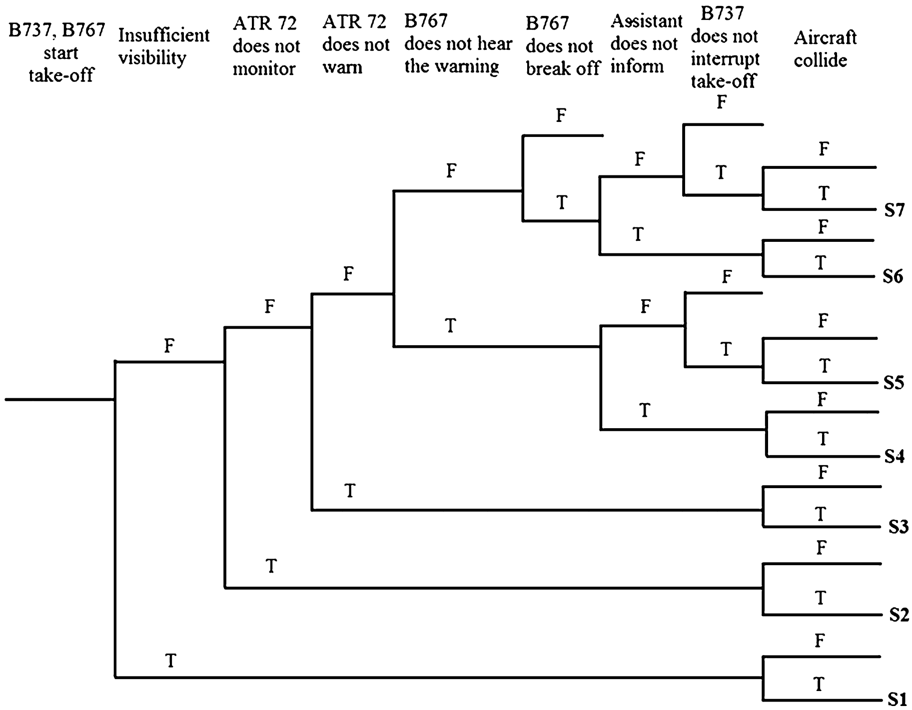

Fig. 1. Event tree presenting scenarios leading to an accident.

Table 1
Scenarios of transformation of Incident 344/07 into an Accident.

	1. Insufficient visibility (E_1)	2. ATR72 does not monitor (E_2)	3. ATR72 does not warn (E_3)	4. B767 does not hear the warning (E_4)	5. B767 does not interrupt take-off (E_5)	6. Assistant controller does not inform controller (E_6)	7. B737 does not interrupt take-off (E_7)	8. Aircraft collide (E_8)
Scenario 1	T	n.r.	n.r.	n.r.	n.r.	n.r.	n.r.	T
Scenario 2	F	T	n.r.	n.r.	n.r.	n.r.	n.r.	T
Scenario 3	F	F	T	n.r.	n.r.	n.r.	n.r.	T
Scenario 4	F	F	F	T	n.r.	T	n.r.	T
Scenario 5	F	F	F	T	n.r.	F	T	T
Scenario 6	F	F	F	F	T	T	n.r.	T
Scenario 7	F	F	F	F	T	F	T	T

used, just because of objections and disagreement among the ICAO experts. However, the only existing numerical probability scale definitions found in the aviation literature are expressed as interval scale. The reason why trapezoidal functions have been selected is because in practice of air traffic management, safety levels are often characterised by disjoint and non-overlapping intervals (Table 2). In fuzzy interpretation such intervals can be graphically expressed by membership functions in the shape of a rectangle. While looking at different memberships functions, trapezoidal functions are best suited to describe this kind of scale. For this reason, triangle membership functions have not been chosen.

A problem remains in how to define parameters of membership functions for values of the linguistic variable *Probability*. Probability scales are used in the analysis of aviation risk, (e.g. as shown in Table 2 [20]). However, because of inconsistencies in the existing scales and a lack of consensus among experts, the authors of this paper have defined a new classification scale.

The linguistic variable *Probability* is shown in Table 3 and in Fig. 2, where it is illustrated by a logarithmic scale. The graphical representation is left side truncated. The reason for selection of a logarithmic scale is that a scale of this type is accepted practice in safety standards. In the first edition of the Safety Management Manual [20], definitions of

Table 2
Definition of probability of occurrence [20].

	Extremely improbable	Very rare	Rare	Probable	Frequent
Qualitative definition	Should virtually never occur in the whole fleet life	Unlikely to occur when considering several systems of the same type, but has to be considered as being possible	Unlikely to occur during the total operational life of each system but may occur several times when considering several systems of the same type	May occur once during total operational life of one system	May occur once or several times during operational life
Quantitative definition	$<10^{-9}$ per flight hour	10^{-7}–10^{-9} per flight hour	10^{-5}–10^{-7} per flight hour	10^{-3}–10^{-5} per flight hour	1–10^{-3} per flight hour

Table 3
Parameters of membership functions of linguistic variable *Probability* values.

	a	*b*	*c*	*d*
λ_{VS}	10^{-20}	10^{-20}	10^{-8}	10^{-7}
λ_{SM}	10^{-8}	10^{-7}	10^{-6}	10^{-5}
λ_{AV}	10^{-6}	10^{-5}	10^{-4}	10^{-3}
λ_{BG}	10^{-4}	10^{-3}	10^{-2}	10^{-1}
λ_{VB}	10^{-2}	10^{-1}	1	1

the probability of air traffic event occurrence are given in a logarithmic scale. Other examples of logarithmic scales are the Tolerable Hazard Rate in railway applications [11], and Safety Integrity Levels in electronic safety related systems [23]. A Logarithmic scale is mentioned by [17], but not analysed. In [42] a case study from a nuclear power plant is presented. Experts' opinions on nuclear event failure possibilities are defined on a linear scale of [0, 1] universe of discourse. Then, using Onisawa's expressions [38], aggregated experts' opinions are transformed into non-fuzzy probability presented on a logarithmic scale.

The variable *Probability* has the following values: very small (*VS*), small (*SM*), average (*AV*), big (*BG*), very big (*VB*). For values *SM*, *AV* and *BG*, trapezoidal membership functions with parameters (a, b, c, d) are as follows:

$$\lambda_i(x; a, b, c, d) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a < x \le b \\ 1, & b < x \le c \\ \frac{d-x}{d-c}, & c < x \le d \\ 0, & x > d \end{cases} \tag{1}$$

where $i \in \{SM, AV, BG\}$.

For values *VS* and *VB*, trapezoidal functions are the following:

$$\lambda_{VS}(x; a, b, c, d) = \begin{cases} 0, & x < a = b \\ 1, & b \le x \le c \\ \frac{d-x}{d-c}, & c < x \le d \\ 0, & x > d \end{cases} \tag{2}$$

$$\lambda_{VB}(x; a, b, c, d) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a < x \le b \\ 1, & b < x \le c = d \\ 0, & x > d \end{cases} \tag{3}$$

Table 2 shows the probability scale for aircraft on-board in-flight system failure. The systems are very reliable. However, as in the analysed incident, unreliability mainly concerns the human factor. Thus in contemporary air traffic systems, human error probability is much higher than aircraft on-board system failure probability.

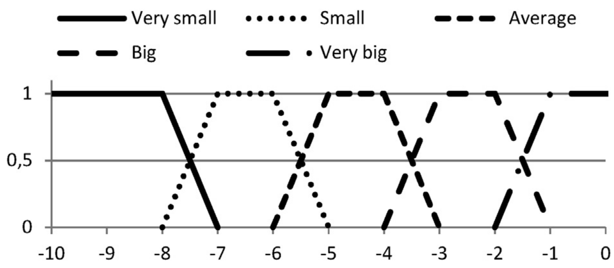

Fig. 2. Linguistic variable *Probability* in logarithmic form.

For commercial aircraft during a flight [12], TLS (Target Level of Safety) is defined at a maximum level of $2.31 \cdot 10^{-8}$ accident probability and this value is the reference point for values *VS* and *SM*. Values *SM*, *AV*, *BG* and *VB* are built on intervals of similar length.

3.2. Logarithmic subsethood measure for determining the value of linguistic variable Probability

In order to calculate the value of linguistic variable *Probability* for fuzzy probability $P(K)$ a similarity measure for two fuzzy sets is required. To find the value of linguistic probabilities for the first type of fuzzy sets Jaccard's similarity measure [24,43], and subsethood measure [32] are used.

Jaccard's measure for fuzzy sets A, B with membership functions λ_A, λ_B is expressed by:

$$s_J(A,B) = \frac{\int_X \min(\lambda_A(x), \lambda_B(x))dx}{\int_X \max(\lambda_A(x), \lambda_B(x))dx} \quad \text{provided } 0 < \int\limits_X \max\bigl(\lambda_A(x), \lambda_B(x)\bigr)dx. \tag{4}$$

Subsethood measure is expressed by:

$$s_S(A,B) = \frac{\int_X \min(\lambda_A(x), \lambda_B(x))dx}{\int_X \lambda_A(x)dx} \quad \text{provided } 0 < \int\limits_X \lambda_A(x)dx. \tag{5}$$

In order to find the value of linguistic probabilities for the second type of fuzzy sets, Jaccard's measure was extended as in [43], and the subsethood measure was extended as in [55]. These measures have a linear nature.

Because the values of linguistic variable *Probability* are given in a logarithmic scale, a measure for determining the value of this variable should be consistent with the intuition of this scale. First, it will be demonstrated that the above two measures are not consistent with the above expectation. Secondly, logarithmic versions of the above two measures for the first type of fuzzy sets will be presented. This logarithmic Jaccard's similarity has been previously used in [34]. The next step is to compare both the logarithmic measures with each other. Advantages of the logarithmic subsethood measure over the logarithmic Jaccard's similarity will be shown by three properties and one theorem.

Considering the proposed logarithmic scale and fuzzy set A expressed by a trapezoidal membership function with parameters $a = 10^{-7}$, $b = 10^{-6}$, $c = 10^{-5}$, and $d = 10^{-4}$. The set and values *SM*, *AV* are given at Fig. 3a).

Assuming measure M has the following property $M(A, SM) = M(A, AV)$, because the picture of A is symmetrically located relative to *SM* and *AV* when compared in a logarithmic scale. However, it is true neither for Jaccard's similarity nor for a subsethood measure. For Jaccard's measure $s_J(A, SM) = 9.01 \cdot 10^{-1}$, $s_J(A, AV) = 9.01 \cdot 10^{-2}$, while for subsethood measure $s_S(A, SM) = 9.09 \cdot 10^{-2}$, $s_S(A, AV) = 9.09 \cdot 10^{-1}$.

The membership function for a linear scale of probability with the universe of discourse $[10^{-20}, 1]$ is:

$$\lambda : \bigl[10^{-20}, 1\bigr] \to [0, 1].$$

The membership function for a logarithmic scale of probability with the universe of discourse $[-20, 0]$ is:

$$\mu : [-20, 0] \to [0, 1].$$

The relations between these functions are as follows:

$$\mu(\log_{10} x) = \lambda(x),$$
$$\mu(x) = \lambda\bigl(10^x\bigr).$$

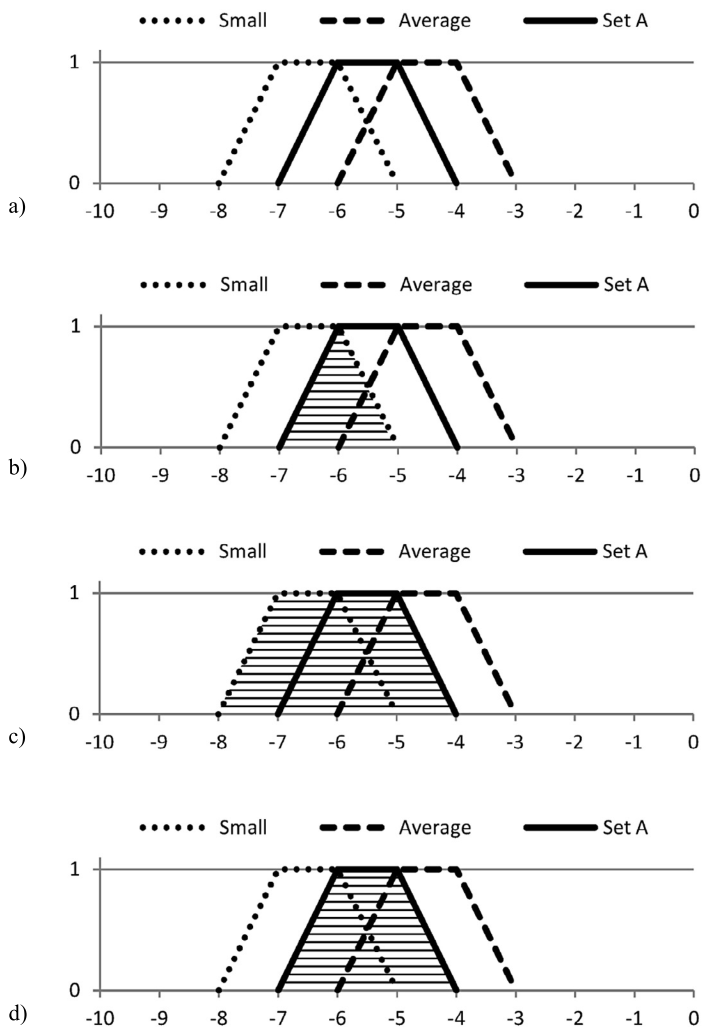

Fig. 3. a) Fuzzy set *A* and values "*Small*" and "*Average*" of linguistic variable *Probability*, b) $\int_{-\infty}^{0} \min(\mu_A(x), \mu_{SM}(x))dx$, c) $\int_{-\infty}^{0} \max(\mu_A(x), \mu_{SM}x)dx$, d) $\int_{-\infty}^{0} \mu_A(x)dx$.

For the membership functions λ_A, λ_B of the fuzzy sets A, B for a linear scale of probability, Jaccard's similarity measure and subsethood measure are given by expressions (4) and (5) with $X = [10^{-20}, 1]$, respectively.

For the membership functions μ_A, μ_B of the fuzzy sets A, B for a logarithmic scale of probability, the logarithmic Jaccard's measure is given by:

$$s_{\log J}(A, B) = \frac{\int_{-20}^{0} \min(\mu_A(x), \mu_B(x))dx}{\int_{-20}^{0} \max(\mu_A(x), \mu_B(x))dx} \quad \text{provided } 0 < \int_{-20}^{0} \max\big(\mu_A(x), \mu_B(x)\big)dx. \tag{6}$$

The logarithmic subsethood measure for fuzzy sets A, B with membership functions μ_A, μ_B for a logarithmic scale of probability is given by:

$$s_{\log S}(A, B) = \frac{\int_{-20}^{0} \min(\mu_A(x), \mu_B(x))dx}{\int_{-20}^{0} \mu_A(x)dx} \quad \text{provided } 0 < \int_{-20}^{0} \mu_A(x)dx. \tag{7}$$

For the above measures and the sets A, SM, some integrals are illustrated in Fig. 3b), c), d).

The following is true:

$$s_{\log J}(A, SM) = 3.33 \cdot 10^{-1}, \qquad s_{\log J}(A, AV) = 3.33 \cdot 10^{-1},$$
$$s_{\log S}(A, SM) = 5 \cdot 10^{-1}, \qquad s_{\log S}(A, AV) = 5 \cdot 10^{-1}.$$

These measures satisfy the requirement $M(A, SM) = M(A, AV)$.

Operations on fuzzy probabilities are executed in a linear scale. The logarithmic subsethood measure is calculated for a logarithmic scale.

Overall procedure for computing the probability of the transformation of an incident into an accident is presented below.

Overall procedure for computing the probability of the transformation of an incident into an accident

1. Construct an Event Tree with fuzzy probabilities for the transformation of an incident into an accident.
2. Calculate membership function λ_A for a linear scale of probability using the Event Tree with fuzzy probabilities developed in step 1.
3. Translate function λ_A into the membership function μ_A.
4. Compute the logarithmic subsethood measure for set A with membership function μ_A.

In the above procedure two truncation operations are required:

A right side truncation is used in summing operations executed on fuzzy probabilities in order to avoid cases where probability would be greater than 1.

The right side truncation is defined by:

$$(a', b', c', d') = rtrunc\big[(a, b, c, d)\big] = \big(\min\{a, 1\}, \min\{b, 1\}, \min\{c, 1\}, \min\{d, 1\}\big)$$

(membership function parameters a', b', c', d' are not greater than 1, i.e. maximal value for linear scale).

The universe of discourse of the membership function μ_A for logarithmic scale of probability is $[-20, 0]$. Let the result of step 3 of the above procedure, i.e. the membership function μ_A be given by (a, b, c, d). In order to have a membership function that satisfies this universe of discourse requirement, the left side truncation is used in the pseudo-code for calculating the logarithmic subsethood measure. This left side truncation is defined by:

$$(a', b', c', d') = ltrunc\big[(a, b, c, d)\big] = \big(\max\{a, -20\}, \max\{b, -20\}, \max\{c, -20\}, \max\{d, -20\}\big)$$

(membership function parameters a', b', c', d' are not smaller than -20, i.e. minimal value of logarithmic scale).

The logarithmic subsethood measure for fuzzy sets A, B, where A is the fuzzy set that value of linguistic variable *Probability* has to be found for, B is a value of linguistic variable *Probability*, with trapezoid membership functions μ_A, μ_B is given by the following pseudo-code.

Pseudo-code for calculating the logarithmic subsethood measure

begin
  $(a_{A1}, b_{A1}, c_{A1}, d_{A1}) = ltrunc[(a_A, b_A, c_A, d_A)]$;
  if $d_{A1} = -20$ **then**
    if $B = VS$ **then** $s_{\log S}(A, B) = 1$ **else** $s_{\log S}(A, B) = 0$;
  if $a_{A1} = 0$ **then**
    if $B = VB$ **then** $s_{\log S}(A, B) = 1$ **else** $s_{\log S}(A, B) = 0$;
  if $0 < \int_{-20}^{0} \mu_{A1}(x)dx$ **then** $s_{\log S}(A, B) = \frac{\int_{-20}^{0} \min(\mu_{A1}(x), \mu_B(x))dx}{\int_{-20}^{0} \mu_{A1}(x)dx}$
end

If $d_{A1} = -20$, then μ_{A1} is the singleton in point -20. Hence, $s_{\log S}(A, B) = 1$ for $B = VS$ and $s_{\log S}(A, B) = 0$ if otherwise. If $a_{A1} = 0$, then μ_{A1} is singleton in point 0. Therefore, $s_{\log S}(A, B) = 1$ for $B = VB$ and $s_{\log S}(A, B) = 0$ if otherwise. If $0 < \int_{-20}^{0} \mu_{A1}(x)dx$ then $s_{\log S}(A, B)$ is calculated according to exp. (7).

Property 1.

$$s_{\log J}(A,B) \le s_{\log S}(A,B) \tag{8}$$

Obvious proof is omitted.

Let A be the fuzzy set for which the value of linguistic variable *Probability* has to be found. Let B be the value of linguistic variable *Probability*.

Let sets A and B membership functions satisfy the following relationships:

$$a_B \le a_A,$$
$$b_B \le b_A,$$
$$c_A \le c_B,$$
$$d_A \le d_B.$$

Therefore, in both linear and logarithmic scales, the picture of set A is contained in the picture of set B. Intuitively, it can be expected that the value of the logarithmic Jaccard's similarity and the logarithmic subsethood measure should be close to 1 because A should have the value B of the linguistic variable. However, only the following is true:

$$s_{\log S}(A,B) = 1 \tag{9}$$

For logarithmic Jaccard's similarity the following holds: $s_{\log J}(A,B) \le 1$. Moreover, let it be additionally proposed that:

$$\int_{-20}^{0} \mu_A(x)dx \ll \int_{-20}^{0} \mu_B(x)dx \tag{10}$$

i.e. the picture of membership function A is included in B's membership function picture, and the surface of the A is much smaller. In such a case, logarithmic Jaccard's similarity has a small value ($\ll 1$) what is inconsistent with the expectation that A should have the value B of the linguistic variable (the similarity should be close to 1).

Considering how the smallest and the greatest values of linguistic variable *Probability* are treated under operations on events.

Consider a more general form of value *VS* (Very small) of other linguistic variables for representing probability in logarithmic scale as given by the following parameters:

$$a_{VS} = -j, \qquad b_{VS} = -j, \qquad c_{VS} = -k, \qquad d_{VS} = -m, \quad \text{provided } m \le k \le j,\ m,k \in N = \{1, 2 \dots\}.$$

Determining the value of linguistic variable *Probability* is supported by left side truncation in the pseudo-code:

$$ltrunc\big[(a,b,c,d)\big] = \big(\max\{a,-j\}, \max\{b,-j\}, \max\{c,-j\}, \max\{d,-j\}\big).$$

This operation guarantees that the result membership function parameters $a_{A1}, b_{A1}, c_{A1}, d_{A1}$ of μ_{A1} in the pseudo-code are not smaller than $-j$.

Property 2. *Let A_1, A_2 be independent events such that the probability of occurrence of each of them is equal to value VS of the linguistic variable. For probability VS^2 of conjunction of these events the following is true:*

$$s_{\log S}\big(VS^2, VS\big) = 1 \tag{11}$$

Proof. Parameters of membership function μ_{A1}, see the pseudo-code, for fuzzy set VS^2 are as follows:

$$a_{VS^2} = b_{VS^2} = \max\{-2j, -j\} = -j, \qquad c_{VS^2} = \max\{-2k, -j\}, \qquad d_{VS^2} = \max\{-2m, -j\}.$$

Hence, $a_{VS^2} = b_{VS^2} = a_{VS} = b_{VS} = -j$, $c_{VS^2} \le c_{VS}$, $d_{VS^2} \le d_{VS}$, and the following holds:

$$\int_{-j}^{0} \min\big(\mu_{VS^2}(x), \mu_{VS}(x)\big)dx = \int_{-j}^{0} \mu_{VS^2}(x)dx. \tag{12}$$

Therefore,

$$s_{\log S}\big(VS^2, VS\big) = 1 \quad \text{provided } 0 < \int_{-j}^{0} \mu_{VS^2}(x)dx. \tag{13}$$

If $d_{VS^2} = -j$, i.e. $\int_{-j}^{0} \mu_{VS^2}(x)dx = 0$ then $s_{\log S}(VS^2, VS) = 1$ according to the pseudo-code.

Considering a more general form of value *VB* (Very big) of other linguistic variables representing probability in logarithmic scale given by the following parameters:

$$a_{VB} = -p, \qquad b_{VB} = -r, \qquad c_{VB} = 0, \qquad d_{VB} = 0, \quad \text{provided } r \le p,\ \ p, r \in N = \{1, 2 \ldots\}.$$

Moreover, j is such that $a_{VS} = b_{VS} = -j$.

The sum operation on fuzzy probabilities is supported by a right side truncation:

$$rtrunc\big[(a, b, c, d)\big] = \big(\min\{a, 0\}, \min\{b, 0\}, \min\{c, 0\}, \min\{d, 0\}\big).$$

This operation guarantees that the result parameters a, b, c, d are not greater than 0. □

Property 3. *Let A_1, A_2 be disjoint events such that the probability of occurrence of each of them is equal to value VB of the linguistic variable. For probability $2 \cdot VB$ of the sum of these events the following is true:*

$$s_{\log S}(2 \cdot VB, VB) = 1 \tag{14}$$

Proof. Parameters of the membership function μ_{A1}, see the pseudo-code, for fuzzy set $2 \cdot VB$ are as follows:

$$a_{2\cdot VB} = \log\big(2 \cdot 10^{-p}\big), \qquad b_{2\cdot VB} = \log\big(2 \cdot 10^{-r}\big), \qquad c_{VB} = d_{VB} = c_{2\cdot VB} = d_{2\cdot VB} = 0.$$

Hence, $a_{VB} < a_{2\cdot VB}$, $b_{VB} < b_{2\cdot VB}$ and the following holds:

$$\int_{-j}^{0} \min\big(\mu_{2\cdot VB}(x), \mu_{VB}(x)\big)dx = \int_{-j}^{0} \mu_{2\cdot VB}(x)dx. \tag{15}$$

Because

$$a_{2\cdot VB} < 0, \quad \text{so } 0 < \int_{-j}^{0} \mu_{2\cdot VB}(x)dx. \tag{16}$$

Hence, $s_{\log S}(2 \cdot VB, VB) = 1$. □

The last two properties are consistent with intuition. These properties do not hold for logarithmic Jaccard's similarity.

Let V denote the set of values of linguistic variable *Probability*.

For the fuzzy set A, $\sum_{B\in V} s_{\log S}(A, B)$ is the sum of logarithmic subsethood measure values over all elements of V.

Theorem 1. *Let A be the fuzzy set to be approximated by a value of linguistic variable Probability, $s_{\log S}(A, B)$ be the logarithmic subsethood measure value for the fuzzy set A, and value B of this linguistic variable, V be the set of values of this variable.*

For the fuzzy set A the following relation is true:

$$1 \leqslant \sum_{B\in V} s_{\log S}(A, B) \leqslant 2 \tag{17}$$

Proof. Let μ_A be the membership function after left truncation. In notation of the pseudo-code it is μ_{A1}.

The main part of the proof is for the case when $0 < \int_{-20}^{0} \mu_A(x)dx$.

$$\sum_{B\in V} s_{\log S}(A,B) = \sum_{B\in V} \frac{\int_{-20}^{0} \min(\mu_A(x), \mu_B(x))dx}{\int_{-20}^{0} \mu_A(x)dx} = \frac{\sum_{B\in V}\int_{-20}^{0} \min(\mu_A(x), \mu_B(x))dx}{\int_{-20}^{0} \mu_A(x)dx}$$
$$= \frac{\int_{-20}^{0}[\sum_{B\in V} \min(\mu_A(x), \mu_B(x))]dx}{\int_{-20}^{0} \mu_A(x)dx} \tag{18}$$

According to Table 3 and Fig. 2, for $x \in [-20, 0]$ there are two alternative cases:

1. there exists one such value $B \in V$ that for $\mu_B(x) = 1$,
2. there exist two such values B_i, B_j that for $k \in \{i, j\}$ the following hold: $0 < \mu_{B_k}(x) < 1$

For Case 1: $\min(\mu_A(x), \mu_B(x)) = \mu_A(x)$.

For Case 2, let the following hold: $\mu_{B_i}(x) \le \mu_{B_j}(x)$.

If $\mu_{B_i}(x) \ge \mu_{B_j}(x)$ then a similar analysis can be executed.

For Case 2 there are three subcases:

a. $\mu_A(x) \le \mu_{B_i}(x)$,
b. $\mu_{B_i}(x) \le \mu_A(x) \le \mu_{B_j}(x)$,
c. $\mu_{B_j}(x) \le \mu_A(x)$.

Therefore, for Case 1: $\sum_{B\in V} \min(\mu_A(x), \mu_B(x)) = \mu_A(x)$.

Hence, for Case 2 for the above subcases, the following holds:

a. $\sum_{B\in V} \min(\mu_A(x), \mu_B(x)) = 2 \cdot \mu_A(x)$,
b. $\mu_A(x) \le \sum_{B\in V} \min(\mu_A(x), \mu_B(x)) \le 2 \cdot \mu_A(x)$,
c. $\sum_{B\in V} \min(\mu_A(x), \mu_B(x)) \le 2 \cdot \mu_A(x)$.

For Subcase 2c, according to Table 3 and Fig. 2, the following relation holds:

$$\forall x \in [-20, 0] \sum_{B\in V} \mu_B(x) = 1. \tag{19}$$

Hence

$$\sum_{B\in V} \min\big(\mu_A(x), \mu_B(x)\big) = \mu_{B_i}(x) + \mu_{B_j}(x) = 1 \ge \mu_A(x) \tag{20}$$

Therefore, when aggregating all cases the following relationship is true:

$$\mu_A(x) \le \sum_{B\in V} \min\big(\mu_A(x), \mu_B(x)\big) \le 2 \cdot \mu_A(x) \tag{21}$$

If $0 < \int_{-20}^{0} \mu_A(x)dx$ then

$$\mu_A(x) \Big/ \int_{-20}^{0} \mu_A(x)dx \le \frac{\sum_{B\in V} \min(\mu_A(x), \mu_B(x))}{\int_{-20}^{0} \mu_A(x)dx} \le 2 \cdot \mu_A(x) \Big/ \int_{-20}^{0} \mu_A(x)dx \tag{22}$$

Next

$$1 = \int_{-20}^{0} \mu_A(x)dx \Big/ \int_{-20}^{0} \mu_A(x)dx \le \frac{\int_{-20}^{0}[\sum_{B\in V} \min(\mu_A(x), \mu_B(x))]dx}{\int_{-20}^{0} \mu_A(x)dx}$$
$$\le \int_{-20}^{0} 2 \cdot \mu_A(x)dx \Big/ \int_{-20}^{0} \mu_A(x)dx = 2. \tag{23}$$

If $d_A = -20$ or $a_A = 0$ then $\sum_{B\in V} s_{\log S}(A, B) = 1$ according to the pseudo-code.

Finally, the thesis is obtained. □

Per the previous discussion it is possible that the logarithmic Jaccard's similarity can be much smaller than 1. It can be found in such a set A that $\sum_{B\in V} \min(\mu_A(x), \mu_B(x)) \ll 1$.

After normalization for logarithmic Jaccard's similarity and logarithmic subsethood measure the following is true:

$$\sum_{B\in V} s_{\log J}(A, B) = 1,$$
$$\sum_{B\in V} s_{\log S}(A, B) = 1.$$

The following holds in all cases that are analysed in this paper using a logarithmic subsethood measure without normalization:

$$\sum_{B\in V} s_{\log S}(A, B) \le 1.12.$$

According to the above considerations, the logarithmic subsethood measure will be used in this paper.

When using interval probabilities instead of fuzzy probabilities, membership functions should satisfy the requirement: $a = b$ and $c = d$. For example, value *Average* of linguistic variable *Probability* is such that $a = b = -5.5$ and $c = d = -3.5$. In such case, the following conclusion holds.

Conclusion 1. Properties 1, 2 and 3 are valid for interval probabilities.

Theorem 1 can be reformulated for interval probabilities to the following shape.

Conclusion 2. Let A be the fuzzy set such that $a = b$ and $c = d$ and to be approximated by a value of the linguistic variable *Probability* with interval probabilities, $s_{\log S}(A, B)$ be the logarithmic subsethood measure value for the fuzzy set A, and value B of this linguistic variable, V be the set of values of this variable.

For the fuzzy set A the following relation is true:

$$\sum_{B\in V} s_{\log S}(A, B) = 1$$

4. Analysis of scenarios potentially leading to an accident

4.1. Analysis of conditions that occurred during the incident

$P_1, P_2, \ldots, P_8$ denote the probability of occurrence of premises $E_1, E_2, \ldots, E_8$ (branching probabilities) conducive to the formation of an aviation accident, and $P(S_1), P(S_2), \ldots, P(S_7)$ denote the probability of realisation of scenarios leading to the transformation of the incident into an accident. Fuzzy probabilities $P_1, P_2, \ldots, P_7$ will be determined on the basis of the literature, analysis of statistical data combined with experts' assessments obtained for this paper. These estimates are not only difficult to obtain but are also subject to a wide margin of error (even if they are of a fuzzy nature and therefore inherently imprecise). A broader discussion of the problems involved in the risk analysis of complex anthropo-technical systems, particularly in relation to air traffic, can be found in [5].

P_1 – the probability that the weather conditions were unfavourable and did not allow incident participants to notice hazards. Determining this probability will be based on an analysis of meteorological data over the last ten years

Table 4
Weather conditions for the Chopin Airport (Weather Underground Internet Service [57]).

	33th week (2005–2014) whole day	33th week (2005–2014) 6.00–22.00	13.08.2007 6.00–22.00
Maximum visibility [km]	30	30	10
Minimum visibility [km]	0.3	1.8	10
Mean visibility [km]	11.8	13.0	10
Visibility less than 500 m [number of observations]	1	0	0
Total number of observations	6720	4480	37
Probability that visibility is less than 500 m	$1.49 \cdot 10^{-4}$	0	0

(2005–2014) for Warsaw Chopin Airport. Observations were considered for the 33rd week (i.e. including 13th Aug.). Since the analysed incident took place in the afternoon, during daylight, the hours 6 am–10 pm were selected to determine the likelihood of P_1. Results of this analysis are shown in Table 4.

The visibility conditions shown in Table 4 are generally very good and there is no interference with airfield observation. In the 10 years analysed, there was not a single case of visibility below 500 m during daylight hours (from 6 am to 10 pm) and it is at a distance of 500 m that the ATR72 crew observed the situation and warned the other participants of the incident. The minimal visibility which was observed over the entire 10 year period is 1800 m, which is much more than the 500 m limit. Taking into account the entire day (including night hours) for the 33rd week of the year over the complete 10 years, there was only one observation of visibility less than 500 m. This was for a situation that lasted for 15 minutes. Therefore one can assume fuzzy probability P_1 as set to "*small*". Obviously, should the analysis take autumn, winter or night conditions into account it would require the adoption of probability P_1 close to the opposite end of the classification scale.

P_2 – the probability that the ATR72 pilot was not observing the runway situation. Under normal conditions, taxiing and take-off preparation is demanding and requires activity focused on these tasks. There is no time available for the observation of the environment. Therefore, as a general case P_2 should be assumed as set to "*very big*". However, in this particular case, waiting in a queue for a take-off (especially if the aircraft is waiting for a long time) allows observation of the environment. In addition, the B737 was to take-off from the same runway as the ATR72, and preceded it on to the taxiway thus, observation by the ATR72 pilot was natural and necessary. The ATR72 also heard the radio communication of all the participants in the event. Therefore, taking the above under consideration, one can assume fuzzy probability P_2 as set to "*big*".

P_3 – describes the probability that the pilot who has noticed the dangerous situation does not communicate what he has noticed. Since professionally trained pilots were involved in the incident, one can assume that fuzzy probability P_3 as set to "*small*".

P_4 – describes the probability that the B767 crew does not pay attention to or does not properly understand the warning communication from the ATR72 pilot. The ATR72 pilot's message did not have to be clear – it could, for instance, merely have indicated the existence of an unusual situation. A somewhat similar probability was estimated in [46], where one of the analysed threats was an undetected warning of a runway occupancy sensor. Therefore, in this paper, one can assume that fuzzy probability P_4 as set to "*big*".

P_5 – describes the probability of failure of the emergency braking manoeuvre. Given the obvious need for this manoeuvre but bearing also in mind the proximity of the speed to the boundary speed v_1 above which an aircraft must continue with take-off, one can assume that fuzzy probability P_5 as set to "*big*."

P_6 – determines the probability of no preventive action being taken by the ATC. Given that the controller was busy with other activities, but also bearing in mind the fact that the controllers most important task is to ensure air traffic safety, the probability of failure to respond to the danger signals P_6 should be considered as set to "*small*".

P_7 – the probability of refusal to execute the controller's command. A conscious refusal seems impossible. However, the B737 crew could either have misunderstood the instruction, or the boundary speed v_1 could have been exceeded. In both these cases an effective response is impossible. Given the above, one can assume that fuzzy probability P_7 as set to "*average*".

P_8 – the probability that continuation of simultaneous take-off results in the accident with fatalities. During aircraft take-off, rapid acceleration takes place and possibilities of manoeuvres to avoid obstacles are extremely limited.

Table 5
Fuzzy probability of premises conducive for the occurrence of an accident.

Premise	Fuzzy probability
E_1 – insufficient visibility (P_1)	*Small* (*SM*)
E_2 – ATR72 does not monitor (P_2)	*Big* (*BG*)
E_3 – ATR72 does not warn (P_3)	*Small* (*SM*)
E_4 – B767 does not hear the warning (P_4)	*Big* (*BG*)
E_5 – B767 does not break off (P_5)	*Big* (*BG*)
E_6 – assistant controller does not inform (P_6)	*Small* (*SM*)
E_7 – B737 does not interrupt take-off (P_7)	*Average* (*AV*)
E_8 – aircraft collide (P_8)	*Very big* (*VB*)

Likewise, if aircraft speed is insufficient, the possibility of ascent to avoid another aircraft is also very limited. In both cases, even if a direct collision is avoided, this does not guarantee that there will be no casualties. Therefore, both these cases are likely to lead to the destruction of at least one aircraft and to deaths as a result of an event other than the direct collision of the two aircraft. Bearing in mind the above, one can assume that fuzzy probability P_8 as set to "*very big*".

All fuzzy probabilities adopted for analysis are shown in Table 5.

Probabilities $P_1, P_2, \dots, P_8$ can be: a value of linguistic variable *Probability* (see Table 5) or a union of some neighbour values of this variable or an aggregation of experts' opinions taken from the set of the linguistic variable values. The probability need to be expressed by a trapezoidal membership function. For example, the union of values *SM* and *AV* is the fuzzy set with trapezoid membership function $\langle a, b, c, d\rangle$, where $a = a_{SM}$, $b = b_{SM}$, $c = c_{AV}$, $d = d_{AV}$, where a_{SM} is the parameter a of the value *SM*.

Probabilities of realisation of the scenarios are as follows:

$$P(S_1) = P_1 \cdot P_8 \tag{24}$$

This is because according to Fig. 1, scenario S_1 is realised if events E_1 and E_8 occur.

$$P(S_2) = (1 - P_1) \cdot P_2 \cdot P_8 \tag{25}$$

because scenario S_2 is realised if E_1 does not occur, while both E_2 and E_8 occur.

$$P(S_3) = (1 - P_1) \cdot (1 - P_2) \cdot P_3 \cdot P_8 \tag{26}$$

$$P(S_4) = (1 - P_1) \cdot (1 - P_2) \cdot (1 - P_3) \cdot P_4 \cdot P_6 \cdot P_8 \tag{27}$$

$$P(S_5) = (1 - P_1) \cdot (1 - P_2) \cdot (1 - P_3) \cdot P_4 \cdot (1 - P_6) \cdot P_7 \cdot P_8 \tag{28}$$

$$P(S_6) = (1 - P_1) \cdot (1 - P_2) \cdot (1 - P_3) \cdot (1 - P_4) \cdot P_5 \cdot P_6 \cdot P_8 \tag{29}$$

$$P(S_7) = (1 - P_1) \cdot (1 - P_2) \cdot (1 - P_3) \cdot (1 - P_4) \cdot P_5 \cdot (1 - P_6) \cdot P_7 \cdot P_8 \tag{30}$$

Let K denote the accident with fatalities, and $P(K)$ the probability of that event occurring which is given by the expression:

$$P(K) = rtrunc\left[\sum_{i=1}^{7} P(S_i)\right] \tag{31}$$

where

$$rtrunc\big[(a, b, c, d)\big] = \big(\min\{a, 1\}, \min\{b, 1\}, \min\{c, 1\}, \min\{d, 1\}\big).$$

The right side truncation operation has been introduced in order to avoid the case that a parameter of membership function for a linguistic variable is greater than 1.

$$P(K) = \big[P_1 + (1 - P_1) \cdot \big(P_2 + (1 - P_2) \cdot \big(P_3 + (1 - P_3) \cdot \big(P_4 \cdot \big(P_6 + (1 - P_6) \cdot P_7\big) + (1 - P_4) \cdot P_5 \cdot \big(P_6 + (1 - P_6) \cdot P_7\big)\big)\big)\big)\big] \cdot P_8 \tag{32}$$

Table 6
Fuzzy probabilities of scenario realisation.

	a	b	c	d
$P(S_1)$	10^{-10}	10^{-8}	10^{-6}	10^{-5}
$P(S_2)$	10^{-6}	10^{-4}	10^{-2}	10^{-1}
$P(S_3)$	$9 \cdot 10^{-11}$	$9.9 \cdot 10^{-9}$	10^{-6}	10^{-5}
$P(S_4)$	$9 \cdot 10^{-15}$	$9.9 \cdot 10^{-12}$	10^{-8}	10^{-6}
$P(S_5)$	$9 \cdot 10^{-13}$	$9.9 \cdot 10^{-10}$	10^{-6}	10^{-4}
$P(S_6)$	$8.1 \cdot 10^{-15}$	$9.8 \cdot 10^{-12}$	10^{-8}	10^{-6}
$P(S_7)$	$8.1 \cdot 10^{-13}$	$9.8 \cdot 10^{-10}$	10^{-6}	10^{-4}
$P(K)$	10^{-6}	10^{-4}	10^{-2}	10^{-1}

Table 7
Values of measure $s_{\log S}$ for fuzzy probability $P(K)$ and values of linguistic variable *Probability*.

Value of linguistic variable *Probability* (*ProbVal*)	$s_{\log S}(P(K), ProbVal)$
Very small (*VS*)	0
Small (*SM*)	$4.8 \cdot 10^{-2}$
Average (*AV*)	$4.3 \cdot 10^{-1}$
Big (*BG*)	$5.7 \cdot 10^{-1}$
Very big (*VB*)	$7.2 \cdot 10^{-2}$

Considering two trapezoidal fuzzy numbers $P_i = (a_i, b_i, c_i, d_i)$ and $P_j = (a_j, b_j, c_j, d_j)$. Their addition, subtraction, multiplication and division respectively, are represented by trapezoidal fuzzy numbers [9,51,53]:

$$(a_i + a_j, b_i + b_j, c_i + c_j, d_i + d_j) \tag{33}$$

$$(a_i - d_j, b_i - c_j, c_i - b_j, d_i - a_j) \tag{34}$$

$$(a_i \cdot a_j, b_i \cdot b_j, c_i \cdot c_j, d_i \cdot d_j) \tag{35}$$

$$(a_i/d_j, b_i/c_j, c_i/b_j, d_i/a_j) \tag{36}$$

In [51] there is an explanation that the above formula for multiplication is an approximation.

When comparing the above approximation with computations based on alfa-cuts, e.g. [10] and [50], the fuzzy arithmetic approximation of the results by trapezoids is less accurate but simpler and less computationally expensive. Until now, there is no agreement about the scale of event probabilities among air transport experts. Hence, the requirement to have more accurate approximation of fuzzy number product than the trapezoid based one, i.e. using the product in (35), does not seem to be strongly motivated.

According to [51], left and right slopes of true fuzzy numbers product are on left when comparing with left and right slopes of trapezoid approximation of the fuzzy numbers product. Therefore, taking into account expression (32) on $P(K)$, this approximation gives greater fuzzy number. Overestimation of probability of occurrence of an accident is the approximation with safety margin.

Fuzzy probabilities of realisation of scenarios $P(S_1), \ldots, P(S_7)$ and fuzzy probability of the accident with fatalities are shown in Table 6.

For trapezoidal fuzzy number $P(K)$ and for each value of linguistic variable *Probability VS*, *SM*, etc., *log S* is given in Table 7.

4.2. Importance of events analysis

In the air traffic incident analysed, none of the premises E_i did in fact occur. However, there is no certainty that this is a permanent condition. Institutions responsible for air traffic safety take many preventive actions to eliminate the factors that contribute to accidents and incidents. The most important questions are however, which factors should be eliminated first and which merit the most attention. For each premise E_i, where $i \in \{1, \ldots, 7\}$, there is a need to find fuzzy probability $P(K|\neg E_i)$ that both aircraft will continue take-off provided that the premise is not true. These

Table 8
Fuzzy probabilities $P(K|\neg E_i)$, where $i \in \{1,\dots,7\}$.

	$P(K\|\neg E_i)$			
	a	b	c	d
$i=1$	10^{-6}	10^{-4}	10^{-2}	10^{-1}
$i=2$	$2\cdot 10^{-10}$	$2.2\cdot 10^{-8}$	$4\cdot 10^{-6}$	$2.2\cdot 10^{-4}$
$i=3$	10^{-6}	10^{-4}	10^{-2}	10^{-1}
$i=4$	10^{-6}	10^{-4}	10^{-2}	10^{-1}
$i=5$	10^{-6}	10^{-4}	10^{-2}	10^{-1}
$i=6$	10^{-6}	10^{-4}	10^{-2}	10^{-1}
$i=7$	10^{-6}	10^{-4}	10^{-2}	10^{-1}

Table 9
Values of measure $s_{\log S}$ for $P(K|\neg E_i)$, where $i \in \{1,\dots,7\}$, and values of linguistic variable *Probability*.

	$s_{\log S}(P(K\|\neg E_i), ProbVal)$				
	VS	*SM*	*AV*	*BG*	*VB*
$i=1$	0	$4.8\cdot 10^{-2}$	$4.3\cdot 10^{-1}$	$5.7\cdot 10^{-1}$	$7.1\cdot 10^{-2}$
$i=2$	$2.9\cdot 10^{-1}$	$4.8\cdot 10^{-1}$	$2.4\cdot 10^{-1}$	$5.3\cdot 10^{-3}$	0
$i=3$	0	$4.8\cdot 10^{-2}$	$4.3\cdot 10^{-1}$	$5.7\cdot 10^{-1}$	$7.1\cdot 10^{-2}$
$i=4$	0	$4.8\cdot 10^{-2}$	$4.3\cdot 10^{-1}$	$5.7\cdot 10^{-1}$	$7.1\cdot 10^{-2}$
$i=5$	0	$4.8\cdot 10^{-2}$	$4.3\cdot 10^{-1}$	$5.7\cdot 10^{-1}$	$7.1\cdot 10^{-2}$
$i=6$	0	$4.8\cdot 10^{-2}$	$4.3\cdot 10^{-1}$	$5.7\cdot 10^{-1}$	$7.1\cdot 10^{-2}$
$i=7$	0	$4.8\cdot 10^{-2}$	$4.3\cdot 10^{-1}$	$5.7\cdot 10^{-1}$	$7.1\cdot 10^{-2}$

fuzzy probabilities would allow for an evaluation of the consequences of preventive activities. According to Table 5, $P(E_8) = VB$, i.e. probability of collision is very high when both aircraft continue take-off. Hence the importance analysis for event E_8 is noted done.

$$P(K \mid \neg E_1) = \big[P_2 + (1-P_2)\cdot\big(P_3 + (1-P_3)\cdot\big(P_4\cdot\big(P_6 + (1-P_6)\cdot P_7\big) + (1-P_4)\cdot P_5\cdot\big(P_6+(1-P_6)\cdot P_7\big)\big)\big)\big]\cdot P_8 \tag{37}$$

$$P(K \mid \neg E_2) = \big[P_1 + (1-P_1)\cdot\big(P_3 + (1-P_3)\cdot\big(P_4\cdot\big(P_6 + (1-P_6)\cdot P_7\big) + (1-P_4)\cdot P_5\cdot\big(P_6+(1-P_6)\cdot P_7\big)\big)\big)\big]\cdot P_8 \tag{38}$$

$$P(K \mid \neg E_3) = \big[P_1 + (1-P_1)\cdot\big(P_2 + (1-P_2)\cdot\big(P_4\cdot\big(P_6 + (1-P_6)\cdot P_7\big) + (1-P_4)\cdot P_5\cdot\big(P_6+(1-P_6)\cdot P_7\big)\big)\big)\big]\cdot P_8 \tag{39}$$

$$P(K \mid \neg E_4) = \big[P_1 + (1-P_1)\cdot\big(P_2 + (1-P_2)\cdot\big(P_3 + (1-P_3)\cdot P_5\cdot\big(P_6+(1-P_6)\cdot P_7\big)\big)\big)\big]\cdot P_8 \tag{40}$$

$$P(K \mid \neg E_5) = \big[P_1 + (1-P_1)\cdot\big(P_2 + (1-P_2)\cdot\big(P_3 + (1-P_3)\cdot P_4\cdot\big(P_6+(1-P_6)\cdot P_7\big)\big)\big)\big]\cdot P_8 \tag{41}$$

$$P(K \mid \neg E_6) = \big[P_1 + (1-P_1)\cdot\big(P_2 + (1-P_2)\cdot\big(P_3 + (1-P_3)\cdot\big(P_4\cdot P_7 + (1-P_4)\cdot P_5\cdot P_7\big)\big)\big)\big]\cdot P_8 \tag{42}$$

$$P(K \mid \neg E_7) = \big[P_1 + (1-P_1)\cdot\big(P_2 + (1-P_2)\cdot\big(P_3 + (1-P_3)\cdot\big(P_4\cdot P_6 + (1-P_4)\cdot P_5\cdot P_6\big)\big)\big)\big]\cdot P_8 \tag{43}$$

Fuzzy probabilities $P(K|\neg E_i)$, as well as for $P(K|\neg E_i)$, where $i \in \{1,\dots,7\}$, and values for linguistic variable *Probability* are given in Tables 8 and 9.

Calculations for the basic variant (Tables 6 and 7) show that the fuzzy likelihood of a scenario with collision is most compliant with a value of *big* (*BG*) for linguistic variable *Probability*.

Analysis of the results of the calculations in Tables 8 and 9 shows that elimination of premises, E_3, E_4, E_5, E_6 and E_7 does not change the fuzzy evaluation of the possibility of transformation of the incident into an accident. The most important premise in this case being E_2 – "the ATR72 does not monitor". It turns out that preventive action was aimed at the elimination of the possibility of this premise moving the evaluation of linguistic variable *Probability* into the area between *average* (*AV*) and *small* (*SM*) values. This means a significant increase in the level of safety. Elimination (or reduction of the likelihood) of premise E_2 is practically possible. Pilot training should therefore be carried out to increase awareness of the need to monitor the airfield during the taxiing procedure and while waiting

Table 10
Values of measure $s_{\log S}$ for other values of P_i, where $i \in \{1, \dots, 8\}$, and for values of linguistic variable *Probability*.

	$s_{\log S}(P(K), ProbVal)$				
	VS	*SM*	*AV*	*BG*	*VB*
$P_1 = VS$	0	$4.8 \cdot 10^{-2}$	$4.3 \cdot 10^{-1}$	$5.7 \cdot 10^{-1}$	$7.1 \cdot 10^{-2}$
$P_1 = AV$	0	$4.8 \cdot 10^{-2}$	$4.3 \cdot 10^{-1}$	$5.7 \cdot 10^{-1}$	$7.2 \cdot 10^{-2}$
$P_2 = AV$	$4.6 \cdot 10^{-2}$	$4.2 \cdot 10^{-1}$	$5.6 \cdot 10^{-1}$	$8.1 \cdot 10^{-2}$	0
$P_2 = VB$	0	0	$5.6 \cdot 10^{-2}$	$5.0 \cdot 10^{-1}$	$5.0 \cdot 10^{-1}$
$P_3 = VS$	0	$4.8 \cdot 10^{-2}$	$4.3 \cdot 10^{-1}$	$5.7 \cdot 10^{-1}$	$7.1 \cdot 10^{-2}$
$P_3 = AV$	0	$4.7 \cdot 10^{-2}$	$4.3 \cdot 10^{-1}$	$5.7 \cdot 10^{-1}$	$7.2 \cdot 10^{-2}$
$P_4 = AV$	0	$4.8 \cdot 10^{-2}$	$4.3 \cdot 10^{-1}$	$5.7 \cdot 10^{-1}$	$7.1 \cdot 10^{-2}$
$P_4 = VB$	0	$4.7 \cdot 10^{-2}$	$4.3 \cdot 10^{-1}$	$5.7 \cdot 10^{-1}$	$7.2 \cdot 10^{-2}$
$P_5 = AV$	0	$4.8 \cdot 10^{-2}$	$4.3 \cdot 10^{-1}$	$5.7 \cdot 10^{-1}$	$7.1 \cdot 10^{-2}$
$P_5 = VB$	0	$4.7 \cdot 10^{-2}$	$4.3 \cdot 10^{-1}$	$5.7 \cdot 10^{-1}$	$7.2 \cdot 10^{-2}$
$P_6 = VS$	0	$4.7 \cdot 10^{-2}$	$4.3 \cdot 10^{-1}$	$5.7 \cdot 10^{-1}$	$7.1 \cdot 10^{-2}$
$P_6 = AV$	0	$4.8 \cdot 10^{-2}$	$4.3 \cdot 10^{-1}$	$5.7 \cdot 10^{-1}$	$7.1 \cdot 10^{-2}$
$P_7 = SM$	0	$4.8 \cdot 10^{-2}$	$4.3 \cdot 10^{-1}$	$5.7 \cdot 10^{-1}$	$7.1 \cdot 10^{-2}$
$P_7 = BG$	0	$4.7 \cdot 10^{-2}$	$4.2 \cdot 10^{-1}$	$5.6 \cdot 10^{-1}$	$7.9 \cdot 10^{-2}$
$P_8 = BG$	$4.2 \cdot 10^{-2}$	$3.7 \cdot 10^{-1}$	$5.0 \cdot 10^{-1}$	$1.7 \cdot 10^{-1}$	$3.9 \cdot 10^{-8}$

for permission to take-off. One could also consider the introduction in operating instructions of a recommendation to carefully observe other traffic.

However, even for the elimination or reduction of the likelihood of premise E_2, the probability $P(K|\neg E_2)$ is still too great.

4.3. Sensitivity analysis

In the importance analysis, for each event the case when the event does not occur is considered. It is often impossible to eliminate a cause. However, sometimes the influence of the cause can be decreased. This means that the value of linguistic variable *Probability* for this cause (event) could be reduced. On the other hand, different experts can present different opinions. Hence, it is worth analysing how the probability for the transformation of the incident into an accident is changed if for an event the other value of occurring probability is selected. For example, if in the analysis performed the value *SM* of event E_i occurring probability is selected then the transformation probabilities for *SM*'s neighbours i.e. *AV* and *VS* are also calculated. The sensitivity analysis consists of this type of analysis.

Results of the sensitivity analysis are given in Table 10. For the row with probability value P_i defined in the left column of the table, the values of other probabilities P_j, where $j \neq i$, are the same as in Section 4.1.

In the previous analysis $P_8 = VB$ and this is the greatest value. Therefore, for P_8 only one value *BG* is considered.

Comparing Tables 7 and 10 the probability $P(K)$ is sensitive only on the values of probabilities P_2 and P_8.

5. Analysis of conditions that are different than in the incident

Other conditions need to be taken into account in order to formulate more general conclusions than those presented in the previous section. To carry out this analysis, real and hypothetical incident cases are divided into four categories according to the incident conditions which are taken into account:

1. Conditions under which the incident has occurred.
2. All the combinations of conditions that could occur (complete analysis).
3. Combinations of conditions that would seem to be the most probable.
4. Combination of the most pessimistic conditions.

Conditions for point 1 have been analysed in the previous section. Thus, in the following three subsections, points 2–4 will be examined.

In each case, the analysis covers:

1. Determination of the values of the input linguistic variables.
2. Calculation of membership functions for linguistic output variables.
3. Logarithmic subsethood measure calculation to find the values of output linguistic variables.

5.1. Complete analysis

In the incident there are eight input linguistic variables (LV's): $P_1, P_2, \dots, P_8$. The output linguistic variables are: $P(S_i)$, $P(K)$, $P(K|\neg E_i)$. Each has five values: *VS*, *SM*, *AV*, *BG*, *VB*.

Introducing a notation for general analysis:

$I(O)$ – set of input (output) linguistic variables,

$|I|$ – the cardinality of the set I, i.e. the number of input LV's,

$i_k \in I - k$-th input LV,

$val(i_k)$ – set of values of the LV i_k, e.g. $\{VS, SM, AV, BG, VB\}$.

n_k – number of values of the LV i_k,

v_{kl} – l-th value of the LV i_k, and $l \in \{\overline{1, n_k}\} = \{1, \dots, n_k\}$,

$\times_{i_k \in I} val(i_k)$ – Cartesian product of the value sets of input LV's.

The complete analysis should be performed for all vectors of values of input LV's:

$$\langle v_{1j}, \dots, v_{kl}, \dots, v_{|I|p}\rangle \in \times_{i_k \in I} val(i_k) \tag{44}$$

and fuzzy probabilities of vector occurrence.

Membership function of output LV $o_p \in O$ is the result of Event Tree with fuzzy probabilities analysis.

Usually, it is very difficult to find fuzzy probabilities for all vectors of input linguistic variable values that are under study and often it is not required to execute such a detailed analysis. This is also the situation with the case study presented in this paper.

5.2. Most probable cases

Considering three cases: in the first $P_1 = AV$ instead of $P_1 = SM$ (as in the incident), in the second case $P_2 = VB$ instead of $P_2 = BG$ (as in the incident), in the third case $P_1 = AV$, $P_2 = VB$.

These estimates take into account both the weather conditions throughout the year (not just during the summer - the case of the incident described in this paper) and the actual amount of traffic at an airport of a similar size to Warsaw Chopin Airport. A more detailed discussion of other possible values of linguistic variables is presented in Section 5.3.

In further analysis, there is only one output linguistic variable $P(K)$.

Case 1

Input LVs:

$$P_1 = AV, \quad P_2 = BG, \quad P_3 = SM, \quad P_4 = BG, \quad P_5 = BG, \quad P_6 = SM, \quad P_7 = AV, \quad P_8 = VB$$

Output LV:

Membership function for $P(K)$

$$a = 10^{-6}, \qquad b = 10^{-4}, \qquad c = 10^{-2}, \qquad d = 10^{-1}.$$

$$s_{\log S}\big(P(K), VS\big) = 0, \qquad s_{\log S}\big(P(K), SM\big) = 4.7 \cdot 10^{-2}, \qquad s_{\log S}\big(P(K), AV\big) = 4.3 \cdot 10^{-1},$$

$$s_{\log S}\big(P(K), BG\big) = 5.7 \cdot 10^{-1}, \qquad s_{\log S}\big(P(K), VB\big) = 7.2 \cdot 10^{-2}.$$

In this case the sought probability is most compliant with values *AV* and *BG*.

Case 2

Input LVs:

$$P_1 = SM, \quad P_2 = VB, \quad P_3 = SM, \quad P_4 = BG, \quad P_5 = BG, \quad P_6 = SM, \quad P_7 = AV, \quad P_8 = VB$$

Table 11
Weather conditions for Warsaw Chopin Airport (Weather Underground Internet Service [57]).

	48th week (2005–2014) whole day	48th week (2005–2014) 6.00–22.00	28.11.2006 6.00–22.00
Maximum visibility [km]	30	30	0.4
Minimum visibility [km]	0.1	0.1	0.1
Mean visibility [km]	6.88	7.3	0.2
Visibility less than 500 m [number of observations]	103	83	37
Total number of observations	6720	4480	37
Probability that visibility is less than 500 m	$1.53 \cdot 10^{-2}$	$1.85 \cdot 10^{-2}$	1

Output LV:
Membership function for $P(K)$

$$a = 10^{-4}, \quad b = 10^{-2}, \quad c = 1.0, \quad d = 1.0.$$

$$s_{\log S}\big(P(K), VS\big) = 0, \quad s_{\log S}\big(P(K), SM\big) = 0, \quad s_{\log S}\big(P(K), AV\big) = 5.6 \cdot 10^{-2},$$

$$s_{\log S}\big(P(K), BG\big) = 5 \cdot 10^{-1}, \quad s_{\log S}\big(P(K), VB\big) = 5 \cdot 10^{-1}.$$

In this case the sought probability is most compliant with values *BG* and *VB*.

Case 3

Input LVs:

$$P_1 = AV, \quad P_2 = VB, \quad P_3 = SM, \quad P_4 = BG, \quad P_5 = BG, \quad P_6 = SM, \quad P_7 = AV, \quad P_8 = VB$$

Output LV:
Membership function for $P(K)$

$$a = 10^{-4}, \quad b = 10^{-2}, \quad c = 1.0, \quad d = 1.0.$$

$$s_{\log S}\big(P(K), VS\big) = 0, \quad s_{\log S}\big(P(K), SM\big) = 0, \quad s_{\log S}\big(P(K), AV\big) = 5.6 \cdot 10^{-2},$$

$$s_{\log S}\big(P(K), BG\big) = 5 \cdot 10^{-1}, \quad s_{\log S}\big(P(K), VB\big) = 5 \cdot 10^{-1}.$$

In this case the sought probability is most compliant with values *BG* and *VB*.

5.3. *Pessimistic condition combination case*

The estimates of the probabilities given in Section 4 have been adopted for conditions as close as possible to the real case which took place in this air incident. However, a similar traffic situation could occur in other, less favourable circumstances. These include: autumn and winter visibility, less experienced pilots etc. Below, pessimistic estimates of the conditions $E_1, \dots, E_8$ are discussed. They express the most unfavourable conditions which realistically could occur.

P_1 – the probability that weather conditions are unfavourable and do not allow incident participants to notice hazards. The most important role in assessing the likelihood of this condition is the visibility impact. Analysis of meteorological data for Warsaw Chopin Airport over the last ten years (2005–2014) shows that the worst minimum visibility is in November, January and February. In terms of the worst week, minimum visibility conditions occur in weeks 48, 47 and 44. To assess the pessimistic value of P_1 data from the 48th week of the year (including November 28th 2006 which is the worst recorded case) were considered. The results of this analysis are shown in Table 11.

Table 11 shows that the minimum visibility observed in the analysed period amounted to 100 m. At the moment of the warning from the ATR72 pilot, the distance between the participants was about 500 m. Thus, the minimum visibility recorded was far too low to notice the danger. In 10 years, during daylight hours (from 6 am to 10 pm), there were as many as 83 cases of visibility below 500 m. Therefore a fuzzy probability P_1 set to "*big*" is assumed.

P_2 – the probability that the ATR72 pilot does not observe the runway situation because taxiing and preparing for take-off is demanding requiring pilots to focus on their own tasks thus leaving no time for observation of the

Table 12
Fuzzy probability of premises conducive to the occurrence of an accident (pessimistic case).

Premise	Fuzzy probability
E_1 – insufficient visibility (P_1)	*Big* (*BG*)
E_2 – ATR72 does not monitor (P_2)	*Very big* (*VB*)
E_3 – ATR72 does not warn (P_3)	*Big* (*BG*)
E_4 – B767 does not hear the warning (P_4)	*Very big* (*VB*)
E_5 – B767 does not break off (P_5)	*Big* (*BG*)
E_6 – Assistant controller does not inform (P_6)	*Small* (*SM*)
E_7 – B737 does not interrupt take-off (P_7)	*Big* (*BG*)
E_8 – aircraft collide (P_8)	*Very big* (*VB*)

environment. In pessimistic case P_2 should be assumed set to "*very big*". The adoption of such estimate may be further justified by the following:

– A queue of planes waiting to take-off may not necessarily occur.
– Taxiing aircraft may have been planned for another runway or a different start point on the same runway. In these situations, visual observation is difficult and not justified by concern for better situational awareness.
– A pilot waiting in a queue may have little experience, making observation difficult.
– The take-off waiting time of the ATR72 could have been very short, which would have meant that radio communication between the pilots of B767 and B737 and the controller would have been during the ATR72 taxiing procedure (not while waiting for take-off). This would have significantly impeded the ATR72 pilot's orientation in the actual traffic situation.

P_3 - describes the probability of the event, that a pilot who spotted the danger does not communicate this information. For an inexperienced pilot, for example, an amateur performing a recreational flight, it is possible that he/she would not be able to properly interpret the threat and thus, would not consider it necessary to inform other users. In these circumstances, it can be assumed fuzzy probability P_3 as set to "*big*".

P_4 - the probability of the event, that the B767 crew does not pay attention or does not properly understand the danger. In a different runway arrangement, especially if runway directions form an obtuse angle, or in situations where objects exist preventing direct observation of an adjacent runway it seems reasonable to assume P_4 probability value as set to "*very big*".

P_7 - the probability of refusal to execute the controller's command. Like probability P_3, a less experienced crew might not understand the seriousness of a particular hazard and thus delay execution of the command. Under these circumstances, a value of "*big*" can be set for fuzzy probability P_7.

All the fuzzy probabilities adopted for pessimistic analysis are shown in Table 12.

Case 4

Input LVs:

$$P_1 = BG, \quad P_2 = VB, \quad P_3 = BG, \quad P_4 = VB, \quad P_5 = BG, \quad P_6 = SM, \quad P_7 = BG, \quad P_8 = VB$$

Output LV:
Membership function for $P(K)$

$$a = 9.1 \cdot 10^{-5}, \quad b = 10^{-2}, \quad c = 1.0, \quad d = 1.0.$$

$$s_{\log S}\big(P(K), VS\big) = 0, \quad s_{\log S}\big(P(K), SM\big) = 0, \quad s_{\log S}\big(P(K), AV\big) = 5.8 \cdot 10^{-2},$$

$$s_{\log S}\big(P(K), BG\big) = 4.9 \cdot 10^{-1}, s_{\log S}\big(P(K), VB\big) = 4.9 \cdot 10^{-1}.$$

In this case the sought probability is most compliant with values *BG* and *VB*.

6. Analysis of accident prevention recommendation prepared by the State Commission for Aircraft Accident Investigation

Risk estimates should always be compared to a risk acceptance level. However in aviation there is no such widely accepted standard. For instance, TLS as proposed by Eurocontrol is only one of a number of proposals. In the probability scale presented in this paper, TLS is "located" between values: "*small*" (*SM*) and "*very small*" (*VS*). In the opinion of the authors, acceptable risk level for Runway Incursion Incidents should be set to *VS*, as it is the only one of the possible events that are considered in TLS. As one can see, according to Section 4.2, even if the likelihood of premise E_2 is reduced or eliminated, the probability $P(K|\neg E_2)$ is too high.

In this section, the main Recommendation of the State Commission for Aircraft Accident Investigation will be analysed. It can be described as follows:

When there are runway crossings, no more than one aircraft can be waiting for permission to take-off on the runway and, as a general principle, waiting should be on the taxiway before the runway threshold.

This Recommendation is directed firstly, at achieving a general elimination of accidents and incidents such as the one analysed and second, to reduce the likelihood of an incident being transformed into an accident. An analysis is presented in this paper to try to assess the effectiveness of this Recommendation by calculating the change in probability of an accident had the Recommendation been implemented (and executed) before Serious Incident 344/07. Reviewing this analysis it should be noted that elimination is not fully possible, as errors can be committed that may lead to a situation where two aircraft are waiting on intersecting runways ready for take-off.

The analysis of the Recommendation is performed in 3 steps:

1. Identifying the probability that despite the Recommendation an accident with fatalities can still take place:
   a) With the same conditions that occurred during the incident.
   b) With the worst cases of the most probable conditions.
   c) With the worst cases of all the possible conditions (pessimistic estimate).
2. Identifying the probability of simultaneous take-off of two aircraft on intersecting runways after the Recommendation has been introduced.
3. Identifying the probability of an accident resulting from an analogous situation to Serious Incident 344/07 but after the Recommendation has been introduced
   a) With the same conditions that occurred during the incident.
   b) With the worst cases of the most probable conditions.
   c) With the worst cases of all the possible conditions (pessimistic estimate).

Step 1.

After introduction of the Recommendation, the probability sought is:

$$P(K_{rec}) = P_9 \cdot P(K) \tag{45}$$

where:

P_9 – fuzzy probability that despite the Recommendation, two aircraft will be waiting for take-off clearance on intersecting runways at the same time.

To ensure comparability of results, the likelihood of the beginning of simultaneous take-off as a result of misinterpretation of air traffic controller commands is not taken into account (similar approach to the one presented in Section 4.1).

The event described with probability P_9 is possible in three cases, the probability of which is denoted as: P_{10}, P_{11} and P_{12}.

P_{10} – fuzzy probability that any of the pilots waiting on the taxiway before the runway threshold, mistakenly understands that their aircraft has received permission to line-up on the runway.

P_{11} – fuzzy probability that the air traffic controller mistakenly gives permission to line-up on the runway to one aircraft waiting on the taxiway before the runway threshold while the other aircraft is already on the second of the intersecting runways.

P_{12} – fuzzy probability that the pilot waiting on the taxiway before the runway threshold, mistakenly understands that their aircraft has received permission to line-up on the runway while in fact it was a take-off clearance given to the aircraft waiting on the second of the intersecting runways.

All these events are independent and disjointed, so the required probability is

$$P(K_{rec}) = (P_{10} + P_{11} + P_{12}) \cdot P(K) \tag{46}$$

The probabilities P_{10}, P_{11} and P_{12} are selected as follows.

P_{10} and P_{12} – Events of this type have been identified in the statistics as causes of air traffic occurrences and are assigned to causal category AGC (Air–Ground Communication). Statistics [13] indicate that about 15% of the events in this category are defined as similar to the analysed case: "wrong aircraft accepts clearance" or "instruction is issued to the wrong aircraft". The same source reports that the incidence of air occurrences from the AGC category is 2.4 events per million instructions issued.

Given the above it can be estimated that the probabilities P_{10} and P_{12} are of the order $0.15 \cdot 2.4 \cdot 10^{-6} = 3.6 \cdot 10^{-7}$, which correspond to "*small*" on the probability scale presented in this paper.

P_{11} – After the implementation of this Recommendation, situations similar to the one described above could only be the result of controller error. This could occur if the controller forgets (for instance), that clearance was already issued for another aircraft. Bearing in mind the fact that this second clearance would have to be issued within a maximum time of a few minutes, probability P_{11} should be set to "*small*".

For the cases: with the same conditions that occurred during the incident, with the worst of the most probable conditions and with the worst of all possible conditions (pessimistic estimate), membership functions for $P(K_{rec})$ are as follows.

a) *With the same conditions that occurred during the incident:*
Input LVs for $P(K_{rec})$:
$P_{10} = SM$, $P_{11} = SM$, $P_{12} = SM$, $P(K)$ is taken from Table 6.
Output LV, membership function for $P(K_{rec})$:

$$a = 3 \cdot 10^{-14}, \quad b = 3 \cdot 10^{-11}, \quad c = 3 \cdot 10^{-8}, \quad d = 3 \cdot 10^{-6}.$$
$$s_{\log S}\big(P(K_{\text{rec}}), VS\big) = 8.2 \cdot 10^{-1}, \quad s_{\log S}\big(P(K_{rec}), SM\big) = 1.9 \cdot 10^{-1},$$
$$s_{\log S}\big(P(K_{rec}), AV\big) = 7 \cdot 10^{-3}, \quad s_{\log S}\big(P(K_{rec}), BG\big) = 0, \quad s_{\log S}\big(P(K_{rec}), VB\big) = 0.$$

In this case the sought probability is most compliant with values *VS* and *SM*.

b) *With the worst cases of the most probable conditions:*
Input LVs for $P(K_{rec})$:
$P_{10} = SM$, $P_{11} = SM$, $P_{12} = SM$, $P(K)$ is taken from subsection 5.2 (case 3).
Output LV, membership function for $P(K_{rec})$:

$$a = 3 \cdot 10^{-12}, \quad b = 3 \cdot 10^{-9}, \quad c = 3 \cdot 10^{-6}, \quad d = 3 \cdot 10^{-5}.$$
$$s_{\log S}\big(P(K_{rec}), VS\big) = 5 \cdot 10^{-1}, \quad s_{\log S}\big(P(K_{rec}), SM\big) = 4 \cdot 10^{-1}, \quad s_{\log S}\big(P(K_{rec}), AV\big) = 1.1 \cdot 10^{-1},$$
$$s_{\log S}\big(P(K_{rec}), BG\big) = 0, \quad s_{\log S}\big(P(K_{rec}), VB\big) = 0.$$

In this case the sought probability is most compliant with values *VS* and *SM*.

c) *With the worst cases of all the possible conditions* (*pessimistic estimate*):
Input LVs for $P(K_{rec})$:
$P_{10} = SM$, $P_{11} = SM$, $P_{12} = SM$, $P(K)$ is taken from subsection 5.3 (case 4).
Output LV, membership function for $P(K_{rec})$:

$$a = 2.7 \cdot 10^{-12}, \quad b = 3 \cdot 10^{-9}, \quad c = 3.1 \cdot 10^{-6}, \quad d = 3.9 \cdot 10^{-5}.$$
$$s_{\log S}\big(P(K_{rec}), VS\big) = 5 \cdot 10^{-1}, \quad s_{\log S}\big(P(K_{rec}), SM\big) = 3.9 \cdot 10^{-1},$$
$$s_{\log S}\big(P(K_{rec}), AV\big) = 1.2 \cdot 10^{-1}, \quad s_{\log S}\big(P(K_{rec}), BG\big) = 0, \quad s_{\log S}\big(P(K_{rec}), VB\big) = 0.$$

In this case the sought probability is most compliant with values *VS* and *SM*.

As it can be noticed, after implementation of the Recommendation of the State Commission for Aircraft Accident Investigation, the probability that an incident similar to the one described could transform into an accident has a value between "*small*" and "*very small*". These are the lowest values in the scale presented in this paper and mean that the Recommendation is also effective in avoiding transformation of the analysed incident into an accident. There is a similar situation with the cases describing the worst of the most probable conditions as well as in the pessimistic condition combination. Moreover, it can be noted that the Recommendation moves accident probability into the zone defined by the TLS value. However, whilst the Recommendation does ensure compliance within current European Standards, it is not effective enough to reach value *VS*.

Step 2.

The analysis in Step 1 shows the effectiveness of the Recommendation in preventing an accident as a result of the occurrence of an incident analogous to 344/07. However, it may also be interesting to analyse the likelihood of repetition of a similar incident after implementation of the Recommendation. This analysis must take into account the additional probability of simultaneous take-off which may be due to many reasons. For the purposes of this paper, the reason considered is the same as in incident 344/07 – erroneous adoption of take-off clearance addressed to another plane. The probability of this event is denoted as P_{13}. It is easy to conclude that its estimation would follow the same reasoning as estimation of probabilities P_{10} and P_{12}. One can assume therefore, that P_{13} is equal to "*small*".

The formula for the probability of the same incident as analysed, but after incorporation of the Recommendation it becomes:

$$P(I) = (P_{10} + P_{11} + P_{12}) \cdot P_{13} \tag{47}$$

Input LVs for $P(I)$:

$$P_{10} = SM, \qquad P_{11} = SM, \qquad P_{12} = SM, \qquad P_{13} = SM.$$

Output LV, membership function for $P(I)$:

$$a = 3 \cdot 10^{-16}, \qquad b = 3 \cdot 10^{-14}, \qquad c = 3 \cdot 10^{-12}, \qquad d = 3 \cdot 10^{-10}.$$

$$s_{\log S}\big(P(I), VS\big) = 1, \qquad s_{\log S}\big(P(I), SM\big) = 0, \qquad s_{\log S}\big(P(I), AV\big) = 0, \qquad s_{\log S}\big(P(I), BG\big) = 0,$$

$$s_{\log S}\big(P(I), VB\big) = 0.$$

In this case the probability sought assumes a value of *VS*. It can clearly therefore be seen, that also in this case the probability reaches a value that is within an acceptable risk level per the scale presented in this paper.

Step 3

After implementation of the Recommendation, the probability of an accident similar to the incident is:

$$P(A) = P(I) \cdot P(K) \tag{48}$$

a) *With the same conditions that occurred during the incident:*
Input LVs:
$P(I)$ is taken from result of (39), $P(K)$ is taken from Table 6.
Output LV, membership function for $P(A)$:

$$a = 3 \cdot 10^{-22}, \qquad b = 3 \cdot 10^{-18}, \qquad c = 3 \cdot 10^{-14}, \qquad d = 3 \cdot 10^{-11}.$$

$$s_{\log S}\big(P(A), VS\big) = 1, \qquad s_{\log S}\big(P(A), SM\big) = 0, \qquad s_{\log S}\big(P(A), AV\big) = 0, \qquad s_{\log S}\big(P(A), BG\big) = 0,$$

$$s_{\log S}\big(P(A), VB\big) = 0.$$

In this case the probability sought assumes a value of *VS*.

b) *With the worst cases of the most probable conditions:*
Input LVs:
$P(I)$ is taken from result of (39), $P(K)$ is taken from subsection 5.2 (case 3).
Output LV, membership function for $P(A)$:

$$a = 3 \cdot 10^{-20}, \quad b = 3 \cdot 10^{-16}, \quad c = 3 \cdot 10^{-12}, \quad d = 3 \cdot 10^{-10}.$$
$$s_{\log S}\big(P(A), VS\big) = 1, \quad s_{\log S}\big(P(A), SM\big) = 0, \quad s_{\log S}\big(P(A), AV\big) = 0, \quad s_{\log S}\big(P(A), BG\big) = 0,$$
$$s_{\log S}\big(P(A), VB\big) = 0.$$

In this case the probability sought also assumes a value of *VS*.

c) *With the worst cases of all the possible conditions* (*pessimistic estimate*):
Input LVs:
$P(I)$ is taken from result of (39), $P(K)$ is taken from Section 5.3 (case 4).
Output LV, membership function for $P(A)$:

$$a = 2.7 \cdot 10^{-20}, \quad b = 3 \cdot 10^{-16}, \quad c = 3.1 \cdot 10^{-12}, \quad d = 3.9 \cdot 10^{-10}.$$
$$s_{\log S}\big(P(A), VS\big) = 1, \quad s_{\log S}\big(P(A), SM\big) = 0, \quad s_{\log S}\big(P(A), AV\big) = 0, \quad s_{\log S}\big(P(A), BG\big) = 0,$$
$$s_{\log S}\big(P(A), VB\big) = 0.$$

In all the cases, implementation of the Recommendation of the State Commission for Aircraft Accident Investigation changes the probability of an incident or an accident of the type analysed to value "*very small*". This is the lowest value in the scale presented in this paper. It means that the Accident Prevention Recommendation is also effective in these cases, and the probability value is within an acceptable risk level per the scale presented in this paper.

7. Results and discussion

This paper presents an analysis of a serious incident which occurred at Warsaw Chopin airport in 2007. Our method is based on expert estimates. Data acquisition session showed considerable amount of their ambiguity. Adoption of interval probabilities would be inconsistent with these observations, particularly in the context of lack of consensus on the probability scale ranges. Therefore, fuzzy probabilities have been used as values of linguistic variable *Probability*. The incident was analysed according to a five value probability scale which is also presented. For the five values appropriate fuzzy sets have been defined. In order to find values of linguistic probabilities, a similarity measure was required and this is also presented. It has also been demonstrated, that for the analysis presented in this paper neither Jaccard's similarity nor subsethood measures for two fuzzy sets are adequate. A new measure named "logarithmic subsethood" has therefore been introduced and its properties have been demonstrated.

Scenarios of transformation of the incident into an accident have been described using Event Trees as well as calculation of fuzzy probability of transformation occurrence. Additionally, it has been shown that elimination of one of the premises for transformation of the incident into an accident significantly reduces the probability of this transformation. However, this reduction is not sufficient.

Sensitivity analysis has been executed using event occurrence probability values which were adjacent to those in the incident. This analysis indicates that the probability of an Air Traffic Incident developing into an Air Traffic Accident is sensitive on the values of occurrence probabilities for two events.

The analysis presented in this paper of the Accident Prevention Recommendation issued by the State Commission for Aircraft Accident Investigation concluded that if this Recommendation is respected by aircraft crew and by air traffic control then the probability of collision occurrence at runway intersections is at the level of the smallest value of linguistic variable *Probability*.

The authors' methodology, developed for incident analysis seems applicable to many Runway Incursions. It enables identification of weak points in airport safety systems as well as the preparation of a ranking of their impact. The approach presented in this paper allows the formulation of post-incident recommendations not only to prevent factors that could led to an incident, but also to strengthen areas that could decrease the probability of the transformation of the incident into an accident.

If a consensus about event probability scale will be elaborated, i.e. interval probabilities of the required scale will be defined, then the proposed approach for interval probabilities with logarithmic subsethood measure, Conclusions 1 and 2 will be applicable.

In this paper, time has been treated qualitatively, i.e. event order is considered without real-time factors. In the authors' next paper on this subject, the time aspect will be treated quantitatively.

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