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Automatic verification of a knowledge base by using a multi-criteria group evaluation with application to security screening at an airport

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ABSTRACT

Knowledge engineering often involves using the opinions of experts, and very frequently of a group of experts. Experts often cooperate in creating a knowledge base that uses fuzzy inference rules. On the one hand, this may lead to generating a higher quality knowledge base. But on the other hand, it may result in irregularities, for example, if one of the experts dominates the others. This paper addresses a research problem related to creating a method for automatic verification of inference rules. It would allow one to detect inconsistencies between the rules that have been developed and the actual knowledge of the group of experts. A method of multi-criteria group evaluation of variants under uncertainty was used for this purpose. This method utilises experts' opinions on the importance of the premises of inference rules. They are expressed in terms of multiple criteria in the form of both numerical and linguistic assessments. Experts define the conclusions of rules as so-called half-marks in order to increase the method's flexibility. Automatic rules are generated in a similar way. Such an approach makes it possible to automatically determine the final conclusions of inference rules. They can be regarded as consistent both with the opinions of a group of experts and with automatically generated rules. This paper presents the use of the method for verifying the rules of an expert system that is aimed to evaluate the effectiveness of a passenger and baggage screening system at an airport. This method allows one to detect simple logical errors that are made when experts are establishing rules as well as inconsistencies between the rules that have been developed and the experts' actual knowledge.

1. Introduction

Knowledge engineering often utilises the opinions of experts. Their participation may be important at different stages of creating an expert system. Among such stages is the development of fuzzy decision rules of a fuzzy inference system that uses rule-based knowledge [17]. The involvement of a group of experts in this process can be particularly useful and effective because this makes it possible to evaluate and interpret elementary facts more adequately, i.e. to avoid unilateral assessments and eliminate simple logical errors.

However, the use of a group of experts' opinions is not without its drawbacks. For example, it is possible that one expert will dominate the others. Personality factors can play a great role during such knowledge gaining sessions. An expert who has extensive knowledge and experience, but who does not have a strong personality can be "convinced" by other experts who are less knowledgeable [27]. This usually happens when experts choose majority voting as a means of reaching a consensus. Then the conclusion of an inference rule may reflect the opinion of the majority more than facts. This situation can be intensified when interdisciplinary problems are investigated, i.e. when particular experts only have competence in certain aspects of the analysed decision-making problem. Therefore they are not able to correctly assess the importance of the other premises that are used in a fuzzy inference system [34,9,32]. Moreover, an expert's knowledge may be inadequately represented in such a system also when a given expert is not willing to present his/her opinion, i.e., for example, in the process of making decisions that are personal in nature [31].

The above-mentioned problems that are associated with using the opinions of a group of experts can lead to creating erroneous rules, which obviously is not advantageous for an expert system that is being developed. Therefore, we propose developing an appropriate model and system for automatic verification of the rules that are generated. Such a system would be aimed not as much at carrying out formal verification as at checking whether the rules that have been obtained correspond to the actual knowledge of a group of experts (although formal verification will also be taken into account). Therefore, it is proposed that an adapted method of multi-criteria group decision-making under uncertainty

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be employed. It will allow the experts to freely (without any pressure or the need to reach a consensus) express their opinions concerning the importance of particular premises of a fuzzy inference system [21]. This approach can also be adopted when one of the methods that facilitate the process of achieving a consensus is used [29,35,28,7,4] because such methods only limit the problems in question during group decision-making, but they do not eliminate them.

1.1. A knowledge base in a fuzzy inference system

Schematically, the fuzzy inference system [20] is presented in Fig. 1.

Non-fuzzy values $\boldsymbol{X}$, obtained through observation or measurements constitute the input of the fuzzification block. In the fuzzification block, based on the specified membership functions, they are associated with the linguistic variables. The fuzzy values $\tilde{\boldsymbol{X}}$ constitute the input for the inference block. This block uses the base of fuzzy rules which in case of our example are created by experts, practitioners in the field of airport security systems. The inference block, on the basis of fuzzy prerequisites and all the fulfilled rules, specifies the conclusion in the form of a linguistic variable $\tilde{y}$. This conclusion is an input for the defuzzification block which on the basis of the specified membership function associates the fuzzy value with the output non-fuzzy value y. It constitutes the result of the operation of the fuzzy inference system.

The rules base may in general contain classic non-fuzzy implications as well as fuzzy implications. In the second case we will use the so called compositional method of reasoning introduced by Zadeh [33] which uses a generalised "modus ponens" fuzzy reasoning rule. This results in the following reasoning scheme [15], where $P, P', Q, Q', S, P_1, P_2$ are fuzzy relations.

$$\begin{aligned} &I : P \Rightarrow Q \\ &F : P' \\ &C : P' \circ (P \Rightarrow Q) \end{aligned} \tag{1}$$

where I denotes implication, F – fact (premise), C – conclusion, while "$\circ$" is a max–min composition, defined on the sets X, Y, Z, whose result for fuzzy relations $A \subset X \times Y$ and $B \subset Y \times Z$ is a fuzzy relation $A \circ B \subset X \times Z$ with a membership function:

$$\mu_{A \circ B}(x, z) = \bigvee_{y \in Y} \left(\mu_A(x, y) \wedge \mu_B(y, z)\right), \quad \forall x \in X,\ \forall y \in Y \tag{2}$$

Relations P and P' are often constructed on the basis of the AND operator. In this case the inference scheme is as follows

$$\begin{aligned} &I : IF\ P_1\ AND\ P_2\ THEN\ Q\ ELSE\ S \\ &F : x_1\ IS\ P'_1\ AND\ x_2\ IS\ P'_2 \\ &C : y\ IS\ Q' \end{aligned} \tag{3}$$

where $P_1, P'_1 \subset X_1, P_2, P'_2 \subset X_2, Q, Q' \subset Y$

Inference result Q' is described in accordance with the compositional inference rule as

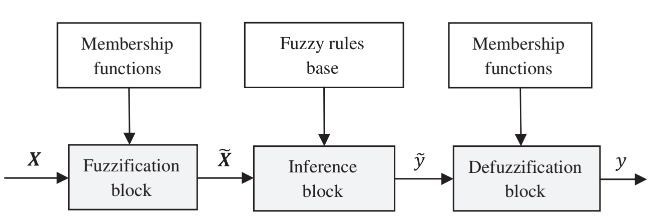

Fig. 1. General structure of the fuzzy inference system.

$$\mu_{Q'}(y) = \bigvee_{(x_1, x_2) \in X_1 \times X_2} \left(\mu_{P'_1}(x_1) \wedge \mu_{P'_2}(x_2) \wedge \mu_R(x_1, x_2, y)\right), \quad \forall y \in Y \tag{4}$$

where $R \subset X_1 \times X_2 \times Y$ is a fuzzy relation described as follows:

$$R = (P_1 \times P_2 \times Q) + (\sim (P_1 \times P_2) \times S) \tag{5}$$

Membership function of a fuzzy relation R may be expressed in the following form

$$\begin{aligned} \mu_R(x_1, x_2, y) = &\left(\mu_{P_1}(x_1) \wedge \mu_{P_2}(x_2) \wedge \mu_Q(y)\right) \\ &\vee \left(\left(1 - \mu_{P_1}(x_1) \wedge \mu_{P_2}(x_2)\right) \wedge \mu_S(y)\right), \\ &\forall (x_1, x_2) \in X_1 \times X_2, \quad \forall y \in Y \end{aligned} \tag{6}$$

Since in the example inference scheme presented in Section 3 the set S does not exist, the membership function has the final form

$$\begin{aligned} \mu_{Q'}(y) = \bigvee_{(x_1, x_2) \in X_1 \times X_2} &\left(\mu_{P'_1}(x_1) \wedge \mu_{P'_2}(x_2) \wedge \left(\mu_{P_1}(x_1) \wedge \mu_{P_2}(x_2) \wedge \mu_Q(y)\right)\right. \\ &\left.\vee \left(1 - \mu_{P_1}(x_1) \wedge \mu_{P_2}(x_2)\right)\right), \quad \forall y \in Y \end{aligned} \tag{7}$$

As for practical applications, like those presented in Section 3 of this paper, the acceptable values of premises $P_1, P_2, \ldots, P_n$ are usually precisely defined. The problem is how to determine the value of the conclusion of rule Q. Experts or groups of experts are often consulted when defining such values. Based on their experience and having shared their views, the experts agree on and formulate clear conclusions for all of the acceptable combinations of premises.

Despite the involvement of a group of experts, there is a danger that certain rules can be constructed incorrectly. There may be simple logical errors as well as inconsistencies related to outranking and the rules may be intransitive, etc. Certainly, the appropriate selection of experts should allow one to avoid such problems, but these cannot be completely eliminated. As for expert systems that are systems of great significance, even several erroneous inference rules can lead to incorrect operation.

1.2. Literature review

Literature that deals with analysing the correctness of expert systems that make use of a knowledge base in the form of fuzzy inference rules mostly focuses on detecting inconsistencies and contradictions in the rules that have been entered into a knowledge base. Detection of such inconsistencies and contradictions is a starting point for removing irregularities. The methods for obtaining and synthesising knowledge that do not allow for creating conflicting rules are analysed to a lesser extent.

Yang et al. [30] proposed using high-level Petri nets to look for common structural errors in expert systems that are based on fuzzy inference rules. The issue of verifying fuzzy inference rules is particularly important for a knowledge base with a small number of rules [14]. Esposito and Maisto [8] attempted to formally verify whether rules were redundant, inconsistent or contradictory. To this end, they used the concept of similarity between fuzzy sets representing the premises and conclusions of inference rules. However, Huang and Cheng [10] emphasise that there are no methods that would allow one to definitely eliminate such problems from a knowledge base. Therefore, they propose an approach that is based on conditional probabilities. The existence of rules that have the same premises but different conclusions is an important type of error that can occur in a fuzzy knowledge base [5,3]. In such situations one can employ a method of direct or priority-based removal of redundant rules [11].

All of these articles are concerned with a case when there might be several rules with the same premises but with different, contradictory conclusions or when there are no rules whatsoever for certain premises in a knowledge base. This situation will not take place here due to the specific manner in which rules will be obtained from experts. Instead, the issue of inconsistency between rules is analysed. The issue of contradictory rules is discussed in a broader context, because it is not as much about the contradictions within a set of rules as the contradictions between the rules and the actual state of knowledge on a given problem.

Viaene et al.'s [26] paper presents a synthesis of methods which attempt to apply algorithms for verifying classical rules of inference in fuzzy inference rules. Both a static and a dynamic approach to verifying fuzzy rules is proposed and the approaches are then compared. Zhang et al. [37,38] elaborated these approaches by adopting a method of detecting a contradictory rule immediately after it has been generated. Ahn and Choi [2] proposed using multi-criteria decision-making methods to solve the problem of conflicting rules in expert systems. In their article they presented conflicting rules, whose conclusions are evaluated in terms of several criteria and as alternatives to choose from. In our paper multi-criteria decision-making is also used, but in a completely different form, since it is assumed that inference rules have several premises and it is these premises (and not conflicting rules) that are alternatives undergoing a multi-criteria group evaluation. This approach allows one to evaluate the importance of a given premise (obtain its weight). This is possible as a result of determining the fuzzy weights of the criteria for evaluating the premises' importance, which also significantly elaborates the approach presented in Ahn and Choi's article [2].

In their paper Huang et al. [12] present the use of particle swarm optimisation in integrating many fuzzy information sources. Thank to this they obtain a knowledge base that would consist of correct rules and that would not be too complicated. The issue of aggregating information in intelligent systems that are based on knowledge represented by fuzzy inference rules was also analysed by Rudas et al. [19]. Abdullah and Amin [1] used generalised fuzzy soft expert set criterion in image encryption applications. Maleszka and Nguyen [16] proposed a method of integrating group knowledge that leads to obtaining a coherent knowledge base. They also pointed out that group knowledge is not a simple sum of knowledge provided by particular group members. Zhang et al. [36] proposed an extended fuzzy multi-criteria group evaluation method which can deal with both subjective and objective criteria for emergency management evaluation. Pei et al. [18] carried out studies on aggregating linguistic assessments, which are also used in the present paper as it also exemplifies this research trend. We adopt a similar approach because our aim is to verify rules before they are built into a knowledge base. However, we also develop these studies by involving a group of experts both in creating the initial rule base and in preparing multi-criteria assessments of particular premises, which make it possible to automatically verify the rules.

The literature review shows that there is a lack of methods that would allow one to automatically check the consistency of a fuzzy rule base that was created by a group of experts with these experts' actual knowledge. While there are analyses that concern the process of obtaining knowledge from a group of experts, such analyses do not focus on the possible irregularities that might occur when a group of cooperating experts arrive at a common position. Yet practice has shown that such irregularities occur very frequently. Therefore, there is a need for creating a method and system for automatic verification of a rule base that was developed by experts. These would make it possible to quickly and effectively identify those rules which, despite being formally correct, do not reflect the experts' actual knowledge.

The main advantage of this method is the ability to freely express opinions by each member of the group of experts. They do not have to negotiate and to agree opinions, which is typical for other methods. The concept of reaching consensus, raised in publications in the field of group decision making, is not always possible or appropriate. In some cases it is preferable that experts evaluate the variants independently (as in the solution proposed here). This is followed by an integration and verification of evaluations using the proposed method. As a result we obtain independent opinions and confidence that the expert assessments were not modified as a result of the pressure from other experts. Another important new feature is the inclusion of the two sources of information into the analysis. More precisely, two different ways to represent preferences. One is the explicit assignment of an evaluation to a combination of input variables. The other one is the determination of evaluation indirectly by defining the importance of criteria and the evaluation of options for these criteria. The use of both of these approaches allows one to automatically verify the fuzzy rules base. This is particularly important if this knowledge base has been created through integration of assessments, including assessments that are not fully compatible. This verification is done using the so-called half-marks, which are also an extension of previous methods.

1.3. Design of the paper

In this paper it is assumed that there is an expert system available and it includes a fuzzy inference system. A set of fuzzy inference rules that takes the following form is the most important element of the system:

$$IF\ x_1 = P_1\ AND\ \dots\ AND\ x_n = P_n\ THEN\ Q \tag{8}$$

where $P_1, P_2, \dots, P_n$ represent a rule's premises and Q denotes a rule's conclusion. It is assumed that the rules (or, in fact, their conclusions) were developed by experts as a result of discussions and negotiations. This paper deals with the issue of verifying these rules. The main research question that is dealt with in the paper is how to automatically verify the correctness of inference rules based on the general preferences stated by experts with regard to premises that are used in a fuzzy inference system. The aim is not to formally verify the correctness of the rules, but mainly to check the extent to which a given rule's conclusion reflects the actual knowledge of the group of experts. Moreover, it is about suggesting to the experts that they rethink particular assessments rather than about defining what the conclusions of specific rules should be.

The structure of this paper is as described below. Section 1 presents the issue of expert systems that utilise fuzzy inference systems and presents a review of literature on verifying decision rules as well as the research problem. Sections 2.1 and 2.2 describe the method of multi-criteria group evaluation of variants under uncertainty and the way in which it was adapted to investigate the problem of determining the importance (weights) of premises in a fuzzy inference system. Section 2.3 contains a description of the way in which particular rules are evaluated by using these weights. Based on the results of this evaluation, automatic rules are generated. Section 2.4. discusses the issue of the flexibility of the method for automatic verification and correction of fuzzy inference rules and proposes that so-called half-marks be used. This section also describes the way in which automatic rules are generated in this case. In section 2.5 an algorithm for choosing the final form of an inference rule's conclusion is presented. Section 3 presents an extensive example of how this method can be used to determine the effectiveness of a passenger and baggage screening system at an airport. Section 4 presents a summary and final conclusions.

2. Automatic verification of inference rules

Problems associated with obtaining knowledge from experts, and in particular from groups of experts, which were mentioned in Section 1, make it necessary to verify the rules that were created by these experts. This can be done by a knowledge engineer who has participated in the process of obtaining knowledge from the experts. In this paper, however, we propose developing a system for automatic verification of rules. This approach has several advantages. First, it is quicker and more effective. Secondly, if the number of rules is large, a knowledge engineer can make the same errors as the experts or may fail to notice certain irregularities. Thirdly, this approach makes it possible to easily carry out another verification if the experts change some of the rules or assessments.

The proposed approach also involves using information obtained from experts for the purpose of the verification. Even though these information items are different than those that are obtained in the process of reaching a consensus about the conclusions of rules and they are collected in a different way, they are still information items obtained from experts. The question then arises of whether the verification information should be provided by the same experts who have developed the rules or from different experts. The first approach allows one to check whether each of the experts' inner conviction about the importance of particular premises was reflected in the rules. The second approach makes it possible to find out if the experts really have clear and unquestionable knowledge. Both of these approaches have advantages. In the present paper the first of the approaches was adopted, which means that both the rules and the verification information were obtained from the same experts. This is because we assumed that we had confidence in the selected experts' knowledge, but we were not completely sure if the process of agreeing on the rule conclusions was correct. It seems that this approach is also more advantageous if one investigates interdisciplinary problems, i.e. when individual experts do not have knowledge of all aspects of a given topic, but only of some aspects of this topic. Another advantage of taking this approach is that the final conclusions of rules that have been proposed by the system are accepted by the experts more easily. This is because they were developed as a result of analysing the opinions that the experts themselves had provided. Hence the problem of accepting other persons' different opinions is eliminated.

2.1. A method of multi-criteria group evaluation of variants under uncertainty

A method of multi-criteria group evaluation of variants under uncertainty forms the basis of the system for automatic verification of a knowledge base that is represented by a set of fuzzy inference rules. In principle, one could employ here any method that leads to the ranking of variants depending on their importance and produces numerical weights which show the level of importance of each of the variants. Given the above, we have decided to employ a method that involves the same experts determining fuzzy inference rules and providing unfettered opinions. They are then used in assessing the importance of particular premises of inference rules. What is more, this evaluation should take into account both objective and subjective criteria. Therefore, the method that is described in Skorupski's article [21] will be used.

The general algorithm of this method is as follows:

1. The group of decision-makers (experts) and variants that are to be evaluated is defined.
2. A set of objective criteria (which make it possible to provide clear, numerical assessments) as well as subjective ones (which require that subjective, linguistic assessments be provided) is determined. This method allows one to take into account both types of criteria at the same time, without the need for artificially reducing the assessments to one type only.
3. The evaluation function for the assessments of variants by the decision-makers for particular (objective and subjective) criteria is determined. Objective assessments are based on values that are represented by real numbers, whereas subjective assessments are based on linguistic values.
4. Decision-makers define the importance of the criteria by making fuzzy assessments. The decision-makers determine the importance of the criteria individually and freely, and they do not have to collaborate with one another.
5. The aggregate weight of the criteria for all of the decision-makers is determined.
6. The assessments of variants are normalised, as a result of which all of the criteria are to be maximised; the evaluation scale is the same. Depending on how the criteria are defined, this stage may be relevant to all of the criteria or to the objective criteria only.
7. A summative linguistic assessment for particular variants for all of the decision-makers is determined.
8. Fuzzy assessments are defuzzified and group preference relations for the variants are constructed.

This algorithm provides a basis for creating an automatic system for verification of rules. The version of the algorithm that has been adapted to the specificity of this problem is described in more detail in Section 2.2. The following sections elaborate the method that is presented in Skorupski's paper [21], which makes it possible to automatically verify the correctness of the inference rules that were proposed by experts.

2.2. Determining the importance (weights) of the premises

It is assumed that the group of experts E participate in defining fuzzy inference rules:

$$E = \{e_i\}, i = 1, \ldots, e \tag{9}$$

The rules' correctness will be verified based on expert opinions; premises P are the input variables of these rules.

$$P = \{p_j\}, \quad j = 1, \ldots, p \tag{10}$$

The essence of the proposed verification is to determine the influence that particular premises have on the conclusion that is drawn from these premises. This influence may vary depending on which aspect of the topic is taken into account. Therefore, the importance of premises will be evaluated with regard to a set of criteria. Among them one can identify objective criteria, i.e. criteria for which one can unambiguously assign a numerical value assessment to particular premises, and subjective criteria, which require an assessment that is linguistic in nature. Therefore, the set of criteria consists of objective criteria:

$$CO = \{c_k\}, \quad k = 1, \ldots, co \tag{11}$$

and of subjective criteria:

$$CS = \{c_k\}, \quad k = co + 1, \ldots, co + cs \tag{12}$$

where co and cs denote the number of the objective criteria and the number of the subjective criteria, respectively.

Evaluation functions for assessing the premises will correspond to the types of criteria: objective ones, i.e.

$$vo : E \times P \times CO \to \mathbb{R} \tag{13}$$

and subjective ones, i.e.

$$vs : E \times P \times CS \to B \tag{14}$$

where $vo(e_i, p_j, c_k), c_k \in CO$ denotes a numerical assessment that was given by the i-th expert to the j-th premise with regard to the k-th (objective) criterion, $vs(e_i, p_j, c_k), c_k \in CS$ denotes a fuzzy linguistic assessment that was given by the i-th expert to the j-th premise with regard to the k-th (subjective) criterion, and $B = \{(x, \mu_B(x)) : x \in X, \mu_B(x) \in [0,1]\}$ denotes a fuzzy set that represents a value of linguistic variable x belonging to the universe of discourse set X, which is described by membership function μ_B.

The next step is to determine the importance of particular criteria for assessing the premises. Many multi-criteria evaluation methods assume that experts (decision-makers) are able to assign numerical values to the weights of the criteria. In reality, this is very difficult to do. Therefore, in practice experts guess rather than really know what the weights of the criteria, or at least the relations between them, are. It is much easier and convenient for experts to assign linguistic assessments to the criteria, such as *important* or *not very important*. It is therefore assumed that the weights of particular criteria take fuzzy values:

$$wg : E \times (CO \cup CS) \to K \tag{15}$$

where

$$K = \{(y, \mu_K(y)) : y \in Y, \mu_K(y) \in [0,1]\} \tag{16}$$

whereas $wg(e_i, c_k) = (y, \mu_K(y)) : y \in Y, \mu_K(y) \in [0,1]$ – denotes a fuzzy linguistic assessment that was given by the i-th expert to the k-th criterion, whereas the universe of discourse set Y denotes the set of the criterion's possible fuzzy weights.

As has already been stated, each of the experts can assign weights $wg(e_i, c_k)$ subjectively and independently. Therefore, it is necessary to calculate the aggregate weight:

$$wga : (CO \cup CS) \to K \tag{17}$$

whereas

$$wga(c_k) = \frac{1}{e}\sum_{i=1}^{e} wg(e_i, c_k) \tag{18}$$

Given that weights $wg(e_i, c_k)$ are linguistic in nature, the form of relation (18) must be determined each time for the assumed form of membership function μ_K. A relevant example is given in Section 3, which presents a possible use of this method in evaluating the effectiveness of a passenger and baggage screening system at an airport.

By using the aggregate weights of the criteria one can determine the summative assessment of the j-th variant by the group of decision-makers:

$$ua(p_j) = \sum_{k=1}^{co}\left(wga(c_k) \cdot \sum_{i=1}^{e} \overline{vo}(e_i, p_j, c_k)\right) + \sum_{k=co+1}^{co+cs}\left(wga(c_k) \cdot \sum_{i=1}^{e} vs(e_i, p_j, c_k)\right) \tag{19}$$

It is also assumed that the universe of discourse set X for subjective criteria is equivalent to interval $[0, x_k]$, where $x_k > 1$; therefore, a summative assessment is made by using normalised evaluation values for objective criteria $\overline{vo}$ which are defined as follows:

$$\overline{vo}(e_i, p_j, c_k) = x_k \cdot \widehat{vo}(e_i, p_j, c_k) \tag{20}$$

where values $\widehat{vo}$ denote assessments that have been normalised to range [0,1] in accordance with the relations:

– for the criteria that are to be maximised:

$$\widehat{vo}(e_i,p_j,c_k) = \frac{vo(e_i,p_j,c_k) - \min_{j=1,\dots,p}(vo(e_i,p_j,c_k))}{\max_{j=1,\dots,p}(vo(e_i,p_j,c_k)) - \min_{j=1,\dots,p}(vo(e_i,p_j,c_k))} \tag{21}$$

– for the criteria that are to be minimised:

$$\widehat{vo}(e_i,p_j,c_k) = \frac{\max_{j=1,\dots,p}(vo(e_i,p_j,c_k)) - vo(e_i,p_j,c_k)}{\max_{j=1,\dots,p}(vo(e_i,p_j,c_k)) - \min_{j=1,\dots,p}(vo(e_i,p_j,c_k))} \tag{22}$$

Just as in formula (18), operations on fuzzy numbers and simultaneous operations on real and fuzzy numbers require to be specified each time for the assumed forms of the membership functions μ_K and μ_B.

Having obtained summative assessments of premises $ua(p_j)$, which are fuzzy sets, we can now determine their weights (importance) by means of defuzzification. In this way non-fuzzy values representing particular fuzzy variables are obtained. In order to rank the premises in terms of importance one can compare fuzzy numbers. But when a method for automatic verification of rules is used, it is exact values that are needed. The best way to obtain such values is through defuzzification. The form of a defuzzification function can be chosen arbitrarily. In general, defuzzification can be performed by using the bisection method. A value $\bar{u}$ that bisects the area under curve μ_B in accordance with the following formula is selected as the value representing fuzzy set:

$$\int_{u_{min}}^{\bar{u}} \mu_B(u)du = \int_{\bar{u}}^{u_{max}} \mu_B(u)du \tag{23}$$

Then, we use $\overline{\overline{ua}}(p_j)$ to denote non-fuzzy values which have been obtained by defuzzification and which represent fuzzy assessments $ua(p_j)$. In this way we obtain the values sought, i.e. weights (importance) of particular premises. It should be noted that experts can freely express their assessments of the criteria as well as of particular premises with regard to specific criteria. The experts do not even have to contact one another, which guarantees them true freedom in making the assessments. Thus, it can be assumed that values $\overline{\overline{ua}}(p_j)$ represent the actual assessments of the importance of particular premises.

2.3. Determining automatic rules by taking into account the premises' weights

By using the aggregate weights of premises that were determined based on experts' unfettered evaluation of the premises' importance, one can determine the correct values of conclusions for all combinations of the premises. According to theory [6], a linguistic variable can be defined as the five-tuple:

$$\langle P, T, X, G, M \rangle \tag{24}$$

where P – a set of names of linguistic variables, which correspond to the premises of fuzzy inference rules in this paper, T – a set of syntactically correct linguistic values of variable P, X – universe of discourse of linguistic variable P, G – syntax of a linguistic variable which is usually expressed through combinatorial grammar and which generates the linguistic values of variable P, and M – semantics of a linguistic variable which is defined by a set of algorithms that make it possible to assign, to each value of a linguistic variable, a certain fuzzy set A, as defined in the universe of discourse X.

Based on linguistic variables that are defined as mentioned above, a set of possible values $T_j \subseteq T$ was determined for each premise $p_j \in P$. Numerical values must be assigned to particular linguistic values. The following function can be used as a starting point:

$$n : P \to \mathbb{N} \tag{25}$$

where $n(p_j)$ denotes the number of linguistic values that premise p_j can take. These values can be arranged in ascending order from the least to the most desirable one, by using the function:

$$pos : P \times T_j \to \mathbb{N} \tag{26}$$

and thus the family of orders can be constructed:

$$R_j = \langle t_r \in T_j : pos(p_j, t_r) = 1, \ldots, t_s \in T_j : pos(p_j, t_s) = n(p_j) \rangle, \quad j = 1, \ldots, p \tag{27}$$

On this basis, the maximum number of values that can be assumed by a linguistic variable is determined:

$$\bar{n} = \max_{j=1,\ldots,p} n(p_j) \tag{28}$$

Value $\bar{n}$ provides a basis for determining the numerical counterparts of particular values of the linguistic variables of premises. This is done by using the function:

$$en : P \times T \to \mathbb{R} \tag{29}$$

where $en(p_j, t_j)$ denotes the numerical value that has been assigned to a value of linguistic variable t_j which is assumed by premise p_j. It should be noted that, for two different premises, two different numerical values can be assigned to the same value of a linguistic variable. These values are assigned in accordance with the relation:

$$en(p_j, t_j) = 1 + \frac{(pos(p_j, t_j) - 1) \cdot (\bar{n} - 1)}{n(p_j) - 1} \tag{30}$$

Relation (30) assigns numerical values so that the least desirable linguistic variable takes the value of 1, the most desirable variable takes the value $\bar{n}$, whereas all of the intermediate values are distributed evenly.

By using the numerical values of particular linguistic variables $en(p_j, t_j)$ and the weights of premises $\overline{\overline{ua}}(p_j)$, we can now carry out the so-called rule evaluation, i.e. define a function for determining the numerical representation of the conclusions of decision rules.

$$ev : T_1 \times T_2 \times \ldots \times T_p \to \mathbb{R} \tag{31}$$

The values of function ev can be defined as follows:

$$ev(t_1, t_2, \ldots, t_p) = \frac{en(p_1, t_1) \cdot \overline{\overline{ua}}(p_1) + en(p_2, t_2) \cdot \overline{\overline{ua}}(p_2) + \ldots + en(p_p, t_p) \cdot \overline{\overline{ua}}(p_p)}{\sum_{j=1}^{p} \overline{\overline{ua}}(p_j)}, \quad t_j \in T_j \tag{32}$$

Assessments that are obtained in this way allow one to divide all rules into sets depending on the assessment of a conclusion. Obviously, the number of these sets must be consistent with the number of the linguistic values that were used by the experts at the stage of formulating the rules. The assignment of a rule to an appropriate set corresponds to defining the rule's conclusion in the form of a linguistic variable, in accordance with the function:

$$ew : W \to \mathbb{R} \times \mathbb{R} \tag{33}$$

where $W = \{w_l\}, l = 1, \ldots, w$ – a set of syntactically correct values of a linguistic variable which is the conclusion of a fuzzy inference rule, whereas values $ew(w_l) = [x_l^1, x_l^2), w_l \in W$, define the lower ($x_l^1$) and the upper ($x_l^2$) limit of the range of values of assessments $ev(t_1, t_2, \ldots, t_p)$, which qualify a rule's conclusion to take linguistic value w_l. If values x_l^1 and x_l^2 are to be determined, first the order of values must be established, analogously to the premises of an inference rule (formula 27). Let us assume that the values are arranged in ascending order, from the least to the most desirable one, and this order is defined as follows:

$$R_c = \langle w_1, w_2, \ldots, w_w \rangle \tag{34}$$

Given these assumptions, values x_l^1 and x_l^2 for linguistic variable $w_l, l = 1, \ldots, w$ can be determined by using these formulas:

$$x_l^1 = 1 + \frac{(l-1)}{w} \cdot \left(\max_{t_j \in T_j} ev(t_1, t_2, \ldots, t_p) - \min_{t_j \in T_j} ev(t_1, t_2, \ldots, t_p) \right) \tag{35}$$

$$x_l^2 = 1 + \frac{l}{w} \cdot \left(\max_{t_j \in T_j} ev(t_1, t_2, \ldots, t_p) - \min_{t_j \in T_j} ev(t_1, t_2, \ldots, t_p) \right) \tag{36}$$

Correct inference rules, which were determined in this way and which are consistent with experts' preferences, can be compared with the inference rules that these experts had agreed on at the initial stage of creating a knowledge base.

2.4. Use of half-marks by experts

The procedure for verifying decision rules that was described in the previous section is relatively restrictive. In practical applications it may point to numerous discrepancies between conclusions that were agreed on by experts and the conclusions that were drawn automatically. On the one hand, this is advantageous because it eliminates any inaccuracies and logical errors that might have been made by the experts. On the other hand, when experts know that the rules they have created are verified by a computer system, this might reduce their identification with the knowledge base that has been developed as well as their interest in correcting the rules. It might make them dissatisfied with the fact that their role as experts is being diminished.

Therefore, we propose adopting a slightly different procedure for obtaining, verifying and establishing the final form of inference rules. This procedure is based on using so-called half-marks. The idea behind this procedure is that half-values, i.e. intermediate values between the established values on the scale, are accepted in the initial version of rule conclusions. For example, if a linguistic variable that describes the conclusion of an inference rule can assume the values {*small*, *average*, *large*} then, if half-marks are to be used, the values {*small*, *small/average*, *average*, *average/large*, *large*} will be accepted. The expression *small/average* should be understood as referring to an intermediate value between *small* and *average*, and it should also be treated as a manifestation of the experts' equal attitude towards both of these marks. The same scale will also be accepted during automatic generation of rules. Therefore, a temporary set (i.e. a set which is being verified) of syntactically correct values of conclusion W_t will be defined as follows:

$$W_1 = \{w_l\}, \quad l = 1, \ldots, w \tag{37}$$

$$W_2 = \{w_l / w_{l+1}\}, \quad l = 1, \ldots, w-1 \tag{38}$$

$$W_t = W_1 \cup W_2 \tag{39}$$

Thus, the order of the linguistic values that describe the conclusions will be as follows:

$$R_w = \langle w_1, w_1/w_2, w_2, \ldots, w_l, w_l/w_{l+1}, w_{l+1}, \ldots, w_w \rangle \tag{40}$$

When the problem is formulated in this way, it is necessary to define the membership criteria for particular assessments, i.e. standard marks and half-marks. Here, the number of different values that can be taken by the conclusion of an inference rule is equal to $2w-1$. Therefore, if new indexes of linguistic values are assumed, i.e.

$$f = 1, 2, \ldots, 2w-1 \tag{41}$$

then values x_f^1 and x_f^2 for linguistic variable $w_f, f = 1, \ldots, 2w-1$ can be determined by using the following formulas:

$$x_f^1 = 1 + \frac{(f-1)}{2w-1} \cdot \left(\max_{t_j \in T_j} ev(t_1, t_2, \ldots, t_p) - \min_{t_j \in T_j} ev(t_1, t_2, \ldots, t_p) \right) \quad (42)$$

$$x_f^2 = 1 + \frac{f}{2w-1} \cdot \left(\max_{t_j \in T_j} ev(t_1, t_2, \ldots, t_p) - \min_{t_j \in T_j} ev(t_1, t_2, \ldots, t_p) \right) \quad (43)$$

The use of half-marks has several advantages. First, this makes it easier to determine the final conclusions of rules if expert and automatic rules are not entirely compatible. The relevant algorithm is presented in Section 2.5. Secondly, such an approach facilitates negotiations between the experts if their assessments are inconsistent. The experts do not have to provide an unambiguous assessment (conclusion), but they can manifest their uncertainty or the lack of agreement within the group by selecting a half-mark. Thirdly, this significantly reduces the possibility of one expert being dominated (outvoted) by the others. A minority voice can be expressed by selecting a half-mark. Fourthly, such a solution promotes greater involvement of experts, who will not feel that they are being replaced by a computer system, but rather that they are only assisted in case of doubt.

2.5. Identifying consistent, inconsistent and undefined rules

In the proposed approach, the process of defining the final form of the conclusions of fuzzy inference rules consists of three stages. At the first stage those conclusions which represent complete or partial consistency between expert and automatic rules and which have only one identical value of a linguistic variable will be selected automatically. At the second stage, where there is complete inconsistency, the experts will be asked to once again think over the conclusion of an inconsistent rule. At the third stage it is necessary that an arbitrary choice be made. This is the case when half-marks are identical, i.e. when both an expert and an automatic conclusion are expressed in the form of identical half-marks. An arbitrary choice between two "half-conclusions" can be made both by the experts and by the automatic system.

Table 1 presents a general diagram of the procedure for selecting the final form of an inference rule.

The entries in the table without resultant conclusions refer to situations in which it is necessary that the experts be consulted again or that the system makes an arbitrary decision. When a rule is inconsistent, this means that the opinion that the experts agreed on is incompatible with these experts' preferences and assessments that they expressed freely, without reaching an agreement. This shows that the experts made an error or that some of them were outvoted (dominated) by others, as a result of which certain opinions were not taken into account when the rules were being defined.

Table 1
Principles of determining a rule's resultant conclusion.

Expert conclusion	Automatic conclusion	Resultant conclusion	Rule type
w_l	w_l	w_l	Consistent
w_l	w_l/w_{l+1}	w_l	Partially consistent
w_l/w_{l+1}	w_l	w_l	Partially consistent
w_{l-1}/w_l	w_l/w_{l+1}	w_l	Partially consistent
w_l	w_{l+1}	None	Inconsistent
w_l/w_{l+1}	w_l/w_{l+1}	None	Undefined

As for an undefined rule, it indicates that both the experts and the automatic verification system have decided that the correct conclusion lies somewhere in between the linguistic values. This is when an arbitrary decision is necessary. This decision can be made both by the experts and by the automatic system.

3. Using the method for verifying inference rules to evaluate the effectiveness of a security screening system at an airport

In order to present the use of this method, we will verify experts' rules concerning the evaluation of the effectiveness of a passenger and baggage screening system at an airport. Security screening at an airport usually consists of three procedures that are aimed to eliminate the instances of carrying on board an aircraft prohibited items, which could be used to commit the so-called act of unlawful interference [13]. Therefore, the following procedures are adopted:

1. passenger screening, which involves screening a passenger by using a walk-through metal detector and, if this causes the detector's activation, also carrying out additional, manual screening;
2. hand baggage screening, which involves screening baggage by using X-ray equipment and then the baggage content being evaluated by a security screener, and also carrying out additional, manual screening if there are any doubts; and
3. checked baggage screening, which involves a multistage evaluation by using automatic equipment, the screened baggage content being evaluated by a security screener, and sometimes also carrying out manual screening in the presence of the passenger.

In our previous papers we analysed the three above-mentioned screening systems as fuzzy inference systems: *Passenger screening*, *Hand baggage* and *Checked baggage*. These systems were partially described in Skorupski and Uchroński's articles [22–24]. In the framework of this research, the impact of various factors (human, organizational and hardware) on the efficiency of the screening system has been examined. This makes it possible to introduce significant changes, for example in the configuration of the security screening checkpoint, technical equipment, procedures used, the selection of employees etc. In all these studies, the fuzzy rules base verification was much easier, because in most cases we are dealing with measurable values. In the final stage there was the need to integrate the knowledge on a very general level. Unfortunately, these data are not measurable. So far, no tools have been provided for objective quality assessment of security control at airports, as we lack the knowledge about the prohibited objects that have been carried on board. Otherwise, we would not have allowed such objects.

The output of each of the three systems is described by a linguistic variable whose membership functions are presented in Fig. 2.

At the last stage of evaluating a passenger and baggage screening system the inference rules for a fuzzy inference system are determined. The three above-mentioned variables, i.e. *Passenger screening*, *Hand baggage* and *Checked baggage*, are the input values of this system. A general evaluation of the effectiveness of a security screening system at an airport is described by the linguistic variable *Evaluation of the system*. Its membership function which is the same as the one presented in Fig. 2, is the output value.

A group of four experts were asked to define the rules. These experts are practitioners with extensive experience in airport security management. The following problem has been brought before them. If we assume that the purpose of the whole security system at the airport is a safe flight (in which there is no explosion,

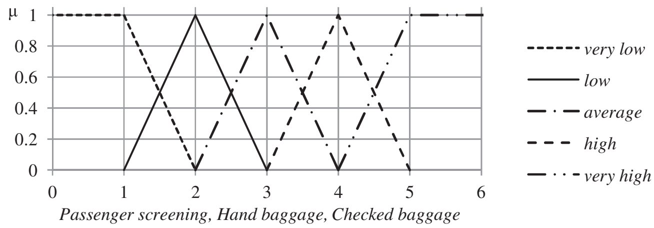

Fig. 2. Membership functions of the linguistic variables: *Passenger screening, Hand baggage, Checked baggage*.

hijacking or an assault on another passenger), which of these three types of screening is the most important to achieve this objective? The experts' evaluations were conflicting. Especially as there are many criteria for the evaluation of these three types of screening. And the importance of these criteria is subjective. This caused the need for integration of assessments, which was the primary motivation to undertake this study.

Thus, in this example:

$$E = \{e_1, e_2, e_3, e_4\}, \quad e = 4 \tag{44}$$

Linguistic variables, which constitute fuzzy premises of the inference rules are:

$$P = \{p_1, p_2, p_3\}, \quad p = 3 \tag{45}$$

where p_1 denotes the variable *Passenger screening*, p_2 denotes the variable *Hand baggage*, and p_3 stands for the variable *Checked baggage*.

In order to determine the importance of these variables (premises) in the final evaluation of a security screening system at an airport, two objective criteria were assumed:

$$CO = \{c_1, c_2\}, \quad co = 2 \tag{46}$$

where c_1 – the estimated number of prohibited items which were allowed to be carried on board an aircraft as a result of a particular type of screening (expressed in percentages) and c_2 – the number of people under threat during the screening itself (the employees at a security screening checkpoint, the passengers who were in this checkpoint's area, technicians, etc.).

A total of eight subjective criteria were adopted:

$$CS = \{c_3, c_4, \ldots, c_{10}\}, \quad cs = 8 \tag{47}$$

These criteria were defined as follows: c_3 – possibility of detecting items that can be used to destroy an aircraft, c_4 – possibility of detecting items that can be used to hijack (take over) an aircraft or terrorise the passengers, c_5 – certainty that each passenger and piece of baggage will be screened, c_6 – possibility of adjusting the scope of screening to the anticipated threat posed by a passenger (profiling), c_7 – flexibility, which is understood as the possibility of narrowing or broadening the scope of screening depending on the level of threat or flight direction, c_8 – resistance to human error, c_9 – resistance to sabotage (for example, when a terrorist is employed as a baggage screener), and c_{10} – capability to adapt to new types of threats.

The experts assigned numerical or fuzzy values to particular premises. Table 2 presents the results of this activity for expert No. 3.

The experts made assessments with regard to all of the subjective criteria by using a linguistic variable that took the values: *low*, *average* and *high*. Let us assume that the fuzzy sets that correspond to particular values of the linguistic variables are given by trapezoidal membership functions, which are described by the universe of discourse set $X = [0, 5]$, given by formulas (48)–(50), whose parameters m, n, p and q are specified in Table 3.

$$\mu_{low}(x; m, n, p, q) = \begin{cases} 0, & x < m = n \\ 1, & n \leqslant x \leqslant p \\ \frac{q-x}{q-p}, & p < x \leqslant q \\ 0, & x > q \end{cases} \tag{48}$$

$$\mu_{average}(x; m, n, p, q) = \begin{cases} 0, & x \leqslant m \\ \frac{x-m}{n-m}, & m < x \leqslant n \\ 1, & n < x \leqslant p \\ \frac{q-x}{q-p}, & p < x \leqslant q \\ 0, & x > q \end{cases} \tag{49}$$

$$\mu_{high}(x; m, n, p, q) = \begin{cases} 0, & x \leqslant m \\ \frac{x-m}{n-m}, & m < x \leqslant n \\ 1, & n < x \leqslant p \\ 0, & x > p = q \end{cases} \tag{50}$$

The next step is to determine the importance of particular criteria for assessing the premises. Let us assume that the experts can assess the importance of the criteria by using fuzzy weights which are described by a linguistic variable that takes the values: *unimportant*, *not very important*, *somewhat important*, *important* and *very important*. These variables will be described by trapezoidal membership functions of fuzzy sets having parameters that are presented in Table 4.

Table 5 presents the weights that were assigned to specific criteria c_k by particular experts e_i.

The next step is to determine the aggregate weights. As discussed in Section 2.2., this step requires defining the meaning of this operation for the adopted membership functions of particular fuzzy sets. Here, trapezoid membership functions were adopted. Therefore, for each of the criteria, the assessment made by the i-th expert can be expressed as follows:

$$(m(wg(e_i, c_k)), n(wg(e_i, c_k)), p(wg(e_i, c_k)), q(wg(e_i, c_k))) \tag{51}$$

where notation $m(wg(e_i, c_k))$ should be interpreted as denoting the first parameter of the trapezoidal membership function for the linguistic variable that was assigned by the i-th expert as a weight to the k-th evaluation criterion. Aggregate weight $wga(c_k)$ will take the form of a trapezoidal membership function with parameters (m^1, n^1, p^1, q^1) which are defined as follows:

$$m^1 = \min_{i=1\ldots e}(m(wg(e_i, c_k))) \tag{52}$$

$$n^1 = \frac{1}{e}\sum_{i=1}^{e} n(wg(e_i, c_k)) \tag{53}$$

$$p^1 = \frac{1}{e}\sum_{i=1}^{e} p(wg(e_i, c_k)) \tag{54}$$

Table 2
Assessments of premises made by expert No. 3: $vo(e_3, p_j, c_k)$, $vs(e_3, p_j, c_k)$.

Criterion	*Passenger screening* (p_1)	*Hand baggage* (p_2)	*Checked baggage* (p_3)
c_1	5	10	15
c_2	30	50	3
c_3	*Average*	*Average*	*High*
c_4	*Average*	*High*	*Low*
c_5	*Average*	*High*	*Low*
c_6	*High*	*High*	*Low*
c_7	*High*	*High*	*Low*
c_8	*High*	*Average*	*Low*
c_9	*Low*	*Average*	*High*
c_{10}	*Average*	*Low*	*Low*

Table 3
Parameters (m, n, p, q) of the trapezoidal membership function for the variables describing the subjective criteria.

	m	n	p	q
Low	0	0	1	2
Average	1	2	3	4
High	3	4	5	5

$$q^1 = \max_{i=1\ldots e}(q(wg(e_i, c_k))) \tag{55}$$

Table 6 presents the aggregate weights for the criteria in the analysed example, which were determined by using formulas (52)–(55).

By taking into account the aggregate weights of the criteria, one can assess the importance of particular premises. However, first it is necessary to normalise the assessments to the same scale and define arithmetic operations on the membership functions of fuzzy sets that have been adopted. For trapezoidal membership functions with parameters (m_1, n_1, p_1, q_1) and (m_2, n_2, p_2, q_2) as well as constant s, these operations will be defined as follows [25]:

$$s \cdot (m_1, n_1, p_1, q_1) = (s \cdot m_1, s \cdot n_1, s \cdot p_1, s \cdot q_1) \tag{56}$$

$$(m_1, n_1, p_1, q_1) \cdot (m_2, n_2, p_2, q_2) = (m_1 \cdot m_2, n_1 \cdot n_2, p_1 \cdot p_2, q_1 \cdot q_2) \tag{57}$$

$$(m_1, n_1, p_1, q_1) + (m_2, n_2, p_2, q_2) = (m_1 + m_2, n_1 + n_2, p_1 + p_2, q_1 + q_2) \tag{58}$$

The assessments of the premises that were normalised in accordance with formulas (20)–(22) with regard to particular criteria for expert 3 are presented in Table 7, whereas the summative assessments for all of the experts, including the aggregate weights of the criteria, are shown in Table 8.

Defuzzification of the assessments is the final step towards implementing the method for assessing the weights of the premises. For the trapezoidal membership functions that have been adopted, defuzzification can be performed by using the bisection method in accordance with the formula:

$$\bar{u} = \frac{1}{4}(m + n + p + q) \tag{59}$$

The obtained results are presented in Table 9.

The defuzzified weights of particular premises of fuzzy inference rules provide a basis for automatic verification of the rules. In this example, the experts defined 125 rules; some of them are presented in Table 10.

In accordance with formula (25), for each premise values $n(p_j)$ are specified; these describe the maximum number of values that can be assumed by linguistic variables. Also, a set of possible values of the variables is defined as well as their order R_j (formula 27). The values for the example that is discussed in this paper are presented in Table 11.

In this example the value of $\bar{n}$ is 5. Example fuzzy inference rules, which are presented in Table 10, were evaluated, as a result

Table 4
Parameters (m, n, p, q) of the trapezoidal membership function for the variables describing the weights of the criteria.

Citerion weight	m	n	p	q
Unimportant	0	0	1	2
Not very important	1	2	3	4
Somewhat important	3	4	5	6
Important	5	6	7	8
Very important	7	8	9	9

Table 5
Weights that were assigned to the criteria by the experts: $wg(e_i, c_k)$.

Criterion number	Expert 1 (e_1)	Expert 2 (e_2)	Expert 3 (e_3)	Expert 4 (e_4)
1 (c_1)	*Very important*	*Very important*	*Important*	*Very important*
2 (c_2)	*Very important*	*Very important*	*Very important*	*Very important*
3 (c_3)	*Important*	*Important*	*Important*	*Very important*
4 (c_4)	*Important*	*Somewhat important*	*Very important*	*Somewhat important*
5 (c_5)	*Important*	*Important*	*Important*	*Important*
6 (c_6)	*Somewhat important*	*Somewhat important*	*Somewhat important*	*Somewhat important*
7 (c_7)	*Important*	*Important*	*Important*	*Important*
8 (c_8)	*Important*	*Somewhat important*	*Important*	*Important*
9 (c_9)	*Very important*	*Important*	*Somewhat important*	*Important*
10 (c_{10})	*Important*	*Important*	*Important*	*Important*

Table 6
Aggregate weights that were assigned to the criteria by the experts.

	$wga(c_k)$
Criterion 1 (c_1)	(5,7.5,8.5,9)
Criterion 2 (c_2)	(7,8,9,9)
Criterion 3 (c_3)	(5,6.5,7.5,8)
Criterion 4 (c_4)	(3,5.5,6.5,9)
Criterion 5 (c_5)	(5,6,7,8)
Criterion 6 (c_6)	(3,4,5,6)
Criterion 7 (c_7)	(5,6,7,8)
Criterion 8 (c_8)	(3,5.5,6.5,8)
Criterion 9 (c_9)	(3,6,7,9)
Criterion 10 (c_{10})	(5,6,7,8)

Table 7
Normalised assessments of the premises made by expert No. 3.

Criterion	*Passenger screening* (p_1)	*Hand baggage* (p_2)	*Checked baggage* (p_3)
c_1	5	2, 5	0
c_2	2.13	0	5
c_3	(1,2,3,4)	(1,2,3,4)	(3,4,5,5)
c_4	(1,2,3,4)	(3,4,5,5)	(0,0,1,2)
c_5	(1,2,3,4)	(3,4,5,5)	(0,0,1,2)
c_6	(3,4,5,5)	(3,4,5,5)	(0,0,1,2)
c_7	(3,4,5,5)	(3,4,5,5)	(0,0,1,2)
c_8	(3,4,5,5)	(1,2,3,4)	(0,0,1,2)
c_9	(0,0,1,2)	(1,2,3,4)	(3,4,5,5)
c_{10}	(1,2,3,4)	(0,0,1,2)	(0,0,1,2)

Table 8
Summative assessment of the premises.

Criterion	*Passenger screening* (p_1)	*Hand baggage* (p_2)	*Checked baggage* (p_3)
c_1	(80 120 136 144)	(37.5 56.25 63.75 67.5)	(45,67.5,76.5,81)
c_2	(14.89 17.02 19.15 19.15)	(34.35 39.26 44.16 44.16)	(140 160 180 180)
c_3	(20 52 90 128)	(20 52 90 128)	(60 104 150 160)
c_4	(12 44 78 144)	(24 66 104 162)	(18 44 78 126)
c_5	(40 72 112 144)	(60 96 140 160)	(35 60 98 128)
c_6	(36 64 100 120)	(36 64 100 120)	(6 16 40 72)
c_7	(60 96 140 160)	(40 72 112 144)	(5 12 42 80)
c_8	(18 55 91 136)	(12 44 78 128)	(18 44 78 112)
c_9	(6 24 56 108)	(24 72 112 162)	(30 84 126 171)
c_{10}	(15 36 70 112)	(10 24 56 96)	(35 60 98 128)
Total	(301.9 580 892.1 1215)	(297.8 585.5 899.9 1212)	(392 651.5 966.5 1238)

Table 9
Results of defuzzification that was performed by using the bisection method.

Criterion	*Passenger screening* (p_1)	*Hand baggage* (p_2)	*Checked baggage* (p_3)
$\overline{\overline{ua}}(p_j)$	747.3	748.7	812

Table 10
Selected fuzzy inference rules that were defined by the experts.

Rule number	*Passenger screening* (p_1)	*Hand baggage* (p_2)	*Checked baggage* (p_3)	Expert conclusion
1	*Very low*	*Low*	*Very low*	*Very low*
7	*Low*	*Satisfactory*	*Very low*	*Very low/low*
43	*Average*	*High*	*Low*	*Average/high*
56	*Very low*	*Satisfactory*	*Average*	*Average*
66	*Very low*	*High*	*Average*	*Average/high*
88	*Average*	*Average*	*High*	*Average*
95	*Very high*	*High*	*High*	*Very high*
98	*Average*	*Very high*	*High*	*Very high*
108	*Average*	*Satisfactory*	*Very high*	*Average*

Table 11
Parameters characterising particular premises.

Parameter	*Passenger screening* (p_1)	*Hand baggage* (p_2)	*Checked baggage* (p_3)
$n(p_j)$	5	5	5
T_j	{*Very low, low, average, high, very high*}	{*Low, satisfactory, average, high, very high*}	{*Very low, low, average, high, very high*}
R_j	⟨*Very low, low, average, high, very high*⟩	⟨*Low, satisfactory, average, high, very high*⟩	⟨*Very low, low, average, high, very high*⟩
$en(p_j, t_j)$	1, 2, 3, 4, 5	1, 2, 3, 4, 5	1, 2, 3, 4, 5

Table 12
Evaluation of selected fuzzy inference rules.

Rule number	*Passenger screening* $en(p_1, t_1)$	*Hand baggage* $en(p_2, t_2)$	*Checked baggage* $en(p_3, t_3)$	$ev(t_1, t_2, t_3)$
1	1	1	1	1.00
7	2	2	1	1.65
43	3	4	2	2.97
56	1	2	3	2.03
66	1	4	3	2.68
88	3	3	4	3.35
95	5	4	4	4.32
98	3	5	4	4.00
108	3	2	5	3.38

Table 13
Membership intervals for particular linguistic variables.

Automatic conclusion (f)	x_f^1	x_f^2
Very low	1	1.4
Very low/low	1.4	1.89
Low	1.89	2.33
Low/average	2.33	2.78
Average	2.78	3.22
Average/high	3.22	3.67
High	3.67	4.11
High/very high	4.11	4.56
Very high	4.56	5

of which a numerical representation of the conclusions of decision rules $ev(t_1, t_2, t_3)$ was obtained. The results of these calculations are presented in Table 12.

Table 14
Comparison (verification) of expert and automatically generated rules.

Rule	Expert conclusion	Result of the evaluation	Automatic conclusion	Resultant conclusion
1	*Very low*	1.0	*Very low*	*Very low*
7	*Very low/low*	1.65	*Very low/low*	*Low*
43	*Average/high*	2.97	*Average*	*Average*
56	*Average*	2.03	*Low*	*Low*
66	*Average/high*	2.68	*Low/average*	*Average*
88	*Average*	3.35	*Average/high*	*Average*
95	*Very high*	4.32	*High/very high*	*Very high*
98	*Very high*	4.0	*High*	*High*
108	*Average*	3.38	*Average/high*	*Average*

In the present example, a method using half-values of the conclusions of fuzzy inference rules was employed. Thus, the order of the linguistic values that describe the conclusions will be defined as follows:

$$R_w = \langle very\ low, very\ low/low, low, low/average, average, average/high, high, high/very\ high, very\ high\rangle \tag{60}$$

Table 13 presents the membership intervals for the conclusions of inference rules which qualify them to be assigned appropriate linguistic values and which were determined by using formulas (42) and (43).

Based on the results of evaluating rules $ev(t_1, t_2, t_3)$, which are presented in Table 12, and the membership intervals that are presented in Table 13, automatic conclusions were generated for the inference rules that are discussed in this example. The results of these calculations are presented in Table 14.

Rule 1 shows that there is complete consistency between an expert and an automatic assessment. Rules 43, 66, 88, 95 and 108 demonstrate partial consistency; in each pair of marks only one value is repeated and it is this value that has been selected as the final conclusion. Rule 7 represents an undefined rule, for which one value must be arbitrarily chosen out of the two values that were accepted both by the experts and the automatic verification system. Rules 56 and 98 represent cases in which the experts' opinion was inconsistent with the rules that were obtained from the verification system. It should be noted that in the real study only 5 out of 125 fuzzy inference rules were inconsistent. When the experts were asked to re-examine their assessments, they quickly came to an agreement and in each case decided in favour of an automatically generated assessment.

4. Summary and final conclusions

The involvement of a group of experts in the process of defining inference rules in fuzzy inference systems provides added value to this procedure, which results from the cooperation and exchange of information between the experts. However, this may also lead to partially inconsistent or misleading results when one of the experts dominates the others. Therefore, it is necessary that the rules created by experts be verified, not only formally, but also in terms of their consistency with these experts' actual knowledge.

A method of multi-criteria group decision-making under uncertainty allows one to determine the importance of premises for the rules that are developed by experts. Given the way in which information is obtained, the weights (importance) of the premises are unambiguous for the group of experts who are consulted. This makes it possible to automatically determine the conclusions of rules for any combination of the premises' values. Automatically generated rules are compared to the rules that were provided by experts in order to detect inconsistencies. As a result of using a new concept of half-marks (both by the experts and the automatic system), it is possible to perform not only the verification, but also automatic selection of the final form of a conclusion. It is consistent both with the experts' assessments and with the automatically generated rules. Inconsistent rules must be re-examined by the experts. In the example that is presented here the experts in each case agreed with the conclusions proposed by the automatic verification system.

In further studies, it would be advisable to investigate the effects of employing one group of experts to formulate the rules and another one to create a verification system. The method that is described in this paper mainly makes it possible to verify whether the rules that have been developed are consistent with the actual preferences of all of the decision-makers. The proposed elaboration of this research topic will rather allow one to check if both groups of experts have the same opinion about the problem. The first approach is more advantageous if one has great confidence in particular experts and when the aim is to check whether the group followed correct principles when formulating the rules. The second approach can be more advantageous if one wants to find out whether the assessments made by a given group of experts were correct.

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