Method of analysis of the relation between serious incident and accident in air traffic

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ABSTRACT: In air transport the last few years resulted in attempts to standardize the methods and tools of risk management, particularly in determining the acceptable (tolerable, target) level of safety TLS. In this paper an original approach of calculating TLS is proposed. Analysis of the various events in air traffic indicates that for serious incidents, there would be sufficient occurrence of only one additional factor, to a serious incident turned into an accident. This observation is the basis for proposing the method, which aims to determine the relationship between serious incident and accident in air traffic. The method uses coloured, timed, stochastic Petri nets for air traffic incidents modelling, as well as for modelling its transformation to accidents. In the paper, extensive example of the serious air traffic incident is described. It shows the opportunities offered by the application of this modelling technique.

1 INTRODUCTION

Air transport is a complex system combining advanced technical systems, operators (air traffic controllers, pilots) and procedures. All these elements work in a large spatial dispersion, but are closely interrelated. They interact, and the time horizon of these interactions is very short. In aviation, the risk is traditionally identified with the air accident, which typically produces a high number of deaths and huge financial losses. Severity of the consequences is the reason why the safety was always a key value in this mode of transport.

Polish aviation regulations define three categories of air events (Aviation Law, 2002):

- accident - as an event during which any person has suffered of serious injuries or aircraft was damaged,
- serious incident - as an incident whose circumstances indicate that there was almost an accident,
- incident - as an event associated with the operation of an aircraft other than an accident, which would adversely affect the safety of operation.

The European Organization for the Safety of Air Navigation (Eurocontrol) issued six documents relating to safety standards, called the ESARR requirements. In air transport the last few years resulted in attempts to standardize the methods and tools of risk management, particularly in determining the acceptable (tolerable, target) level of safety (Skorupski, 2009). Currently, European aviation authorities use safety minimums set by the ECAC (European Civil Aviation Conference) denoting an acceptable level of safety only for the category of "accidents". TLS (Target Level of Safety) defines the maximum value of probability of an accident, for the commercial aircraft, to be equal $1{,}55 \cdot 10^{-8}$ accident on a flight hour, or $2{,}31 \cdot 10^{-8}$ accident on a flight (Eurocontrol, 2001).

All ECAC member states are obliged to designate the so called CLS (Current Level of Safety) and compare it to TLS. This task is substantially difficult because the TLS concept is based on the number of accidents with regard to the volume of traffic. In many countries, however, there have been no air accidents in recent years. In this case, a reliable determination of the required CLS value is impossible.

One method of solving this problem is to use data on air incidents, which are obviously more frequent than air accidents. If the value of the CLS determined based on incidents is within the limits specified for the accident, it is assumed that this is a satisfactory result, not requiring further research or action (Dong-bin et al., 2009).

In this paper a different approach is proposed. Serious air traffic incidents should be analysed and used to determine the probability of transforming them into accidents. A method for modelling incidents with use of coloured, stochastic, timed Petri nets, with the time assigned to markers is described.

The first part of the paper presents basic information about Petri nets. The next discusses an algorithm of the method. The next chapter contains an example of analysis using the proposed method.

2 THE BASICS OF PETRI NETS

Petri nets provide a convenient way to describe many types of systems. Especially a lot of applications they found in software engineering, where they are used particularly to describe and analyze concurrent systems. There is a rich literature in this subject, e.g. (Szpyrka, 2008), which also contains an extensive bibliography of the topic.

Depending on the needs, one can define different Petri nets with certain properties. The basis for building a Petri net is a bipartite graph containing two disjoint sets of nodes called places and transitions. Below are presented brief definitions of basic types of Petri nets (Marsan et al. 1999).

2.1 *Generalised Petri net*

Generalised Petri net (GPN) is described as:

$$N = \{P, T, I, O, H\} \tag{1}$$

where: P - set of places,
T - set of transitions, $T \cap P = \emptyset$,
I, O, H, are functions respectively of input, output and inhibitors:
$I, O, H: T \rightarrow B(P)$
where $B(P)$ is the superset over the set P.
Given a transition $t \in T$ it can be defined:
$t^+ = \{p \in P: I(t,p) > 0\}$ - input set of transition t
$t^- = \{p \in P: O(t,p) > 0\}$ - output set of transition t
$t^o = \{p \in P: H(t,p) > 0\}$ - inhibition set of transition t.

2.2 *Marked Petri net*

Marked Petri net (MPN) is described as:

$$S_M = \{P, T, I, O, H, M_0\} \tag{2}$$

where: $N = \{P, T, I, O, H\}$ - generalised Petri net,
$M_0: P \rightarrow \mathbb{Z}_+$ is the initial marking. We also say that the marking specifies the number of markers assigned to each of the places.

Transition t is called active in marking M, if and only if:

$$\forall p \in t^+, M(p) \geq I(t,p) \wedge \forall p \in t^o, M(p) < H(t,p) \tag{3}$$

Firing of transition t, active in marking M, removes from any place p belonging to the set t^+, as many markers as function $I(t,p)$ determines. At the same time it adds to any place p from the set t^-, as many markers as determined by the $O(t,p)$ function. This means firing of transition t will change actual marking to M' such that

$$M' = M + O(t) - I(t) \tag{4}$$

This relationship is written briefly $M[t\rangle M'$. We then say that M' is reachable directly from M. If the $M \rightarrow M'$ transformation requires firing a sequence of transitions σ, then we say that M' is reachable from M and denote $M[\sigma\rangle M'$.

2.3 *Place-transition Petri net*

Place-transition net (PTN) is supplemented by the characteristics of places interpreted as their capacity. Thus, a place-transition net can be written as

$$S_{PT} = \{P, T, I, O, H, K, M_0\} \tag{5}$$

where: $N = \{P, T, I, O, H\}$ - generalised Petri net,
$K: P \rightarrow \mathbb{N} \cup \{\infty\}$ – capacity of places, and the symbol ∞ means that a place has unlimited capacity,
$M_0: P \rightarrow \mathbb{Z}_+ \wedge \forall p \in P: M_0(p) \leq K(p)$ – initial marking.

2.4 *Timed Petri net*

With timed Petri net (TPN) we have to do, when firing a transition is not immediate, but it takes a certain time:

$$S_T = \{P, T, I, O, H, M_0, \tau\} \tag{6}$$

where $S_M = \{P, T, I, O, H, M_0\}$ – marked Petri net,
$\tau: T \rightarrow \mathbb{R}_+$ – delay function, specifying static delay $\tau(t)$ of transition t.

Characteristics on transitions may determine time associated with firing of the transition in different ways. In particular, this value may be described by a deterministic or a random variable. In the latter case, we may say that a network is stochastic. In addition to static delay it is sometimes convenient to use dynamic delay $\delta(t)$, defined as the rest of the time remaining until the firing of the transition t.

2.5 *Coloured Petri net*

In coloured Petri nets (CPN) there is ability to define markers of different types. Marker type is called a colour. Each place in the coloured net is assigned a set of colours that it can store. Expressions are assigned to arcs and transitions that allow manipulating various types of markers. Coloured Petri net can be written as

$$S_C = \{\Gamma, P, T, I, O, H, C, G, E, M_0\} \tag{7}$$

where $S_M = \{P, T, I, O, H, M_0\}$ – marked Petri net,
Γ – nonempty, finite set of colours,
C – function determining what colour markers can be stored in a given place: $C: P \rightarrow \Gamma$,
G - function defining the conditions that must be satisfied for the transition, before it can be fired; these are the expressions containing variables belonging to Γ, for which the evaluation can be made, giving as a result a Boolean value,
E – function describing the so-called weight of arcs, i.e. expressions containing variables of types belonging to Γ.

2.6 *Coloured, timed Petri net*

It is possible to combine the idea of CPN and TPN. In this case the following structure of coloured, timed Petri net (CTPN) is formed

$$S_{CT} = \{\Gamma, P, T, I, O, H, C, G, E, M_0, R, r_0\} \tag{8}$$

where: $S_M = \{P, T, I, O, H, M_0\}$ – marked Petri net,
Γ – nonempty, finite set of colours, each of which can be timed, that means whose elements are pairs consisting of colour and a timestamp,
C, G, E – have the same meaning as in the case of CPN, but taking into account the fact that certain sets of colours can be timed,
R - set of timestamps (also called time points), closed under the operation of addition, $R \subseteq \mathbb{R}$,
r_0 – initial time, $r \in R$.

2.7 *Petri nets properties*

For each Petri net we can determine among others: reachability graph, reachability set and liveness. In the presented method of analysis, the most important property of the net (which model a traffic incident) is the reachability of selected states (markings) from initial marking M_0. It allows assessing the probability and time of reaching those selected markings.

Particularly important are the dead markings, because they illustrate the situations in which we can assess whether the traffic process results in an incident or in an accident.

3 METHOD OF ANALYSIS OF THE RELATION BETWEEN SERIOUS AIR TRAFFIC INCIDENT AND ACCIDENT

Preliminary analysis of the various events in air traffic indicates that for the events classified as serious incidents, there would be sufficient occurrence of only one additional conducive factor, or the termination of only one inhibiting factor, to a serious incident turned into an accident. This observation is the basis for proposing the following method of analysis.

3.1 *Purpose and scope of the analysis*

While analyzing risk of serious incidents, with the use of event tree or fault tree, there are many elements which probabilities we do not know. In addition, events are dependent (in the probabilistic sense). That makes analysis more difficult.

In the literature we find many attempts to determine probability of an accident, also using Petri nets. The most advanced is risk analysis method TOPAZ (Shortle et. al. 2003). This method allows a qualitative analysis of the process leading to the accident. Obtaining quantitative results is difficult and uncertain because of the need to include a large number of assumptions and constraints related to the environment of the system investigated.

The method presented in this paper is based on analyzing only those additional factors that determine the creation of the accident. This definitely reduces the scope of analysis and also reduces the uncertainty of risk estimation. At the same time, this approach is adequate to achieve the goals of analysis - to determine the statistical dependencies between a serious incident and the accident. As a result of finding such a relationship, it is possible to estimate the number of accidents just on the basis of knowledge of the number of incidents.

3.2 *Outline of proposed method*

In the method presented in this paper the following interpretation was adopted:

- The set of places P corresponds to traffic situations. These situations are referred to both the location of a plane in the airspace, as well as to issue of specific permits (clearances). The set P may include, for example, situations such as: aircraft ready for take-off, occupied runway, the plane at the intersection of the runways, taxiing started, etc. Additional elements of this set are situations describing the state of the environment, such as: the occurrence of more than 1000 meters of visibility, ATC controller busy, the pilot of another aircraft watches the situation on the manoeuvring area etc.

  Traffic situation may involve a single aircraft, which is modelled by the occurrence of a single marker in a place $p \in P$. It can also affect many aircraft simultaneously. Then the number of aircraft is modelled by more markers ($M(p) > 1$). Since the traffic situation refers to a particular airspace, which has limited capacity, therefore places are typically characterized by a capacity function K. The same is the interpretation of places that model the states of environment. For example, if a place corresponds to the absence of visibility, then the occurrence of several markers can model the fact that several participants of traffic (controllers, pilots, car drivers) are not able to observe the situation.
- The set of transitions T corresponds to the set of events (actions) that change the traffic situation, particularly affecting the safety of manoeuvres. These are events such as: ATC controller allows the start, the plane taxiing at a certain taxiway, the plane does not stop the actual manoeuvre. These events can be characterized by two values: the time of their duration (including the important role played by the zero-time events, the so-called immediate events) and a priority, defined by the probability of realization of events that can occur simultaneously.

Activation of transition is dependent on an adequate number of markers in input places. It may also depend on the occurrence of certain additional conditions, which define the time at which the markers appear in the input places or the probabilities of events described by the relevant random variables. Activation and firing rules for transitions determine the dynamics of the modelled incident.

- The input function *I* defines the traffic situations that determine occurrence of certain events, output function *O* defines what event (action) must occur to change the status of the analyzed system, and the inhibitor function *H* specifies the traffic situations that must not exist to certain events can occur.

  In many cases, the functions *I* and *O* take the values greater than 1, which means that for the occurrence of certain events, not only the existence of specific traffic situation is necessary, but also the fact that a specific number of objects is involved in it. That is indicated by a sufficient number of markers in input places. This property can be modelled through the use of place-transition Petri nets.
- The initial marking M_0 defines the traffic situation in which we begin the analysis, and the current marking *M* describes the current state of the system (process).

  If the exact timing plays an important role in transforming the incident into an accident, the marking must be accompanied by the time stamp for each marker. The occurrence of different types of objects (e.g. aircraft of different weight category) can determine the use of CPN, and thus markers of different colours. In these cases, traffic situations (places of the net) must also be characterized by the colour of marker.

The analysis, which aims to determine the relationship between serious incident and accident in air traffic, can be realized in two ways:

1 On the basis of relevant Petri nets, modelling the actual incident and the hypothetical accident, we can determine reachability graph. Its analysis allows the analytical determination of the searched probability.

2 Simulation of the process (serious incident, accident) modelled by a suitable Petri net, together with recording of time and probability of staying in each state. In particular, it might be interesting to observe the frequency of reaching one of the dead markings that correspond to the transition from incident to accident. This frequency will match the searched probability (Skorupski 2010).

3.3 *Algorithm of the method*

General algorithm of the method is as follows:

1 Development of a model of serious air traffic incident as a Petri net. It is necessary to take into account all the events (leading to or inhibiting the incident) and time relations between them. Definition of the proper sequence of events can be obtained by defining the duration of events (and transitions corresponding to them) as a deterministic value, related to the real duration of these events. Another possibility is the use of inhibitor arcs, which do not allow to fire a transition before the required traffic situation will not end.

2 Reduction of the network, which consists in elimination of places and transitions that do not affect the transformation of the incident into accident. Arcs adjacent to those nodes are also removed. This stage of the method allows for significant reduction in the number of states considered in subsequent stages.

3 Development of the scenarios transforming an incident into accident. These scenarios must take into account both: the appearance of additional events and absence of inhibiting events. Selection of scenarios is very important in the method. They should include: hardware failures, wrong decisions of the controller and the pilot, refusal to comply with the recommendations of the controller, lack of situational orientation, bad weather conditions etc. On the other hand, one should not extend the area of analysis beyond the factors closely related to the accident under consideration.

4 Development of a model of an accident, taking into account reduction of the network and all the possible scenarios as defined in previous section. This step requires determination of nodes and arcs that are added to the net. For newly added elements it is necessary to determine their characteristics: for places - capacity, colour, initial marking; for transitions - conditions of firing, execution time, the way a dynamic delay $\delta(t)$ is treated; for arcs - weights and characteristics depended on the type of network (especially for coloured nets). During this stage, one should also consider which network elements are stochastic, and take into account relevant characteristics of random variables of transition execution time. This refers primarily to randomization of events time sequence (mentioned in step 1 of the algorithm). Apart from inclusion of random component to transition execution time, it is necessary to remove most of inhibitor arcs. The last part of this step of the algorithm is to define the initial marking M_0.

5 Determination of reachability set and reachability graph of developed Petri net. This stage of the algorithm is also part of the model validation process. It is necessary to examine whether, for all dead states, it is possible to definitively determine that there has been a transformation of the

incident into an accident or not. In the latter case we are dealing with a situation demanding a return to section 4 of the algorithm and re-analysis of network structure in order to eliminate errors.

6 Reduction of reachability graph. This graph often contains dozens or even hundreds of states, which makes it difficult to analyze. Then, there is a need for its reduction. The issue is difficult and complicated. First, the methods for the reduction recommended by the literature can be used: reduction using symmetry and reduction related to stable sets of transitions. The second reduction step may be to remove states that occur with probability of 1, and do not affect the results of the study. This is a reduction due to specific purpose of analysis. A side effect of the reduction process can be a proof that simplification of the initial Petri net structure is possible. This happens when certain sequences of states are always performed, without the possibility of choosing a different sequence while firing transitions. One can then go back to step 4 of this algorithm and modify (simplify) the structure of the network.

7 Isolation of system states representing the transformation of the incident into accident (final states). Depending of the adopted type of Petri net, its structure, but also of the used software tool - states referred to, may be living states (stable or vanishing) or dead. It is recommended to structure the model in such a way, that it is not possible to continue firing transitions when it is already known, that in the course of the simulation an accident occurred, or that the transformation of the incident into an accident is impossible. In other words, it is recommended to achieve a situation where final states of the analysis, are the dead states.

8 An analytical or simulative determination of total probability of accident. For each of the final states there are usually many sequences, which lead to them from the initial marking. For easier finding of those sequences we can extract the partial subgraph of reachability graph, containing all the paths leading from M_0 to the analyzed final state. In addition, this subgraph must contain all the states reachable directly from any state included in these paths. This will allow analyzing the probability of passing these sequences, or alternatively - the probability of leaving such a sequence. The sum of probabilities of various sequences give the probability of reaching given final state. And the total for all final states, gives the probability of transforming an incident into accident. In a situation where there is a lot of paths, or they consist of big number of states - analysis is difficult and not very effective. In that case, using a simulative method is recommended. It consists in the iterative execution of simulation experiments, the outcome of which is dependent on the weights of immediate transitions and of realization of random variables describing the duration of the transition execution time. The frequency of reaching final states is the sought probability of transforming the incident into an accident. Additionally, we should note here, that the partial subgraph isolation, can allow for the continuation of graph reduction process, in the same way as it was mentioned in section 6 of this algorithm.

4 EXAMPLE ANALYSIS – SERIOUS AIR TRAFFIC INCIDENT 344/07

As an example illustrating the method a serious air traffic incident, which occurred in August 2007 at Warsaw airport will be presented. Its participants were Boeing 767 and Boeing 737 aircraft, and its cause was classified as a "human factor" and the causal group H4 - "procedural errors" (Civil Aviation Authority, 2009).

4.1 *Circumstances of the serious incident*

In the incident on 13th of August 2007 two aircraft – Boeing 737 (B737) and the Boeing 767 (B767) participated. They more or less at the same time were scheduled for take-off from the Warsaw Chopin airport. As the first, clearance for line-up and wait on runway RWY 29 was issued to B737. As a second, clearance for line-up and wait on runway RWY 33 was given to B767 crew. The latter aircraft was the first to obtain permission to take-off. A moment after confirmation of permission to take-off, both aircraft began start procedure at the same time. Situation was extremely dangerous, as runways are crossing at this airport. B737 crew assumed that the start permission was addressed to them. They probably thought that since they first received permission to line up the runway, they are also the first to be permitted to start. In addition, the categories of wake turbulence caused, that it would be better to start B737 before B767, from the traffic point of view. Decision of the controller, however, was different. An air traffic controller (ATC) did not watch planes take-off, because at this time he was busy agreeing helicopter take-off. The situation of simultaneous start was observed by the pilot of ATR 72, which was standing in queue for departure. He reacted on the radio. After this message, B767 pilot looked right and saw B737 taking-off. Then, on his own initiative, broke off and began a rapid deceleration, which led to stopping the plane 200 meters from the intersection of the runways. Assistant controller heard the ATR 72 pilot radio message and informed the controller that B737 operate without authorization. A controller, who originally did not hear the information by radio, after 16 seconds from the start, recognized the situation and strongly ordered B737 to discontinue take-off procedure. B737 crew performed braking and stopped 200 m from the intersection of the runways.

4.2 *Model of serious incident*

This air traffic incident almost led to collision between the two aircraft, it means to accident. As in most such situations, there were many factors contributing to the creation of this dangerous situation. The most important are:

- lack of situational awareness at the B737 crew,
- inadequate monitoring of radio communications and, consequently, wrong acceptance of permission for the start, in fact directed to another plane,
- lack of the crew cooperation in the B737 cockpit,
- lack of proper monitoring of the take-off by the controller,
- controller's lack of response to the information from the pilot of ATR 72 transmitted by radio.

The factors impeding the development of the accident, which resulted in preventing it, include:

- good assessment of dangerous situation by the crew of B767 and decision to immediately discontinue take-off,
- good recognition of the hazard by the crew of the ATR 72 and immediate sending a message by radio,
- good weather conditions for visual observation of the runways,
- proper response of assistant controller.

TPN model representing this serious incident is shown in Figure 1.

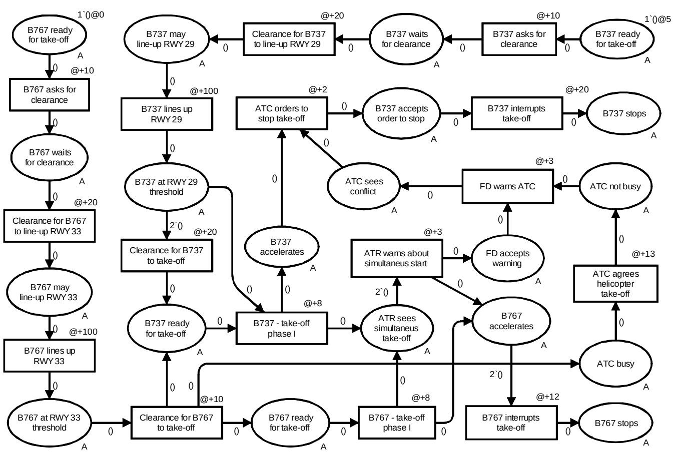

Figure 1. The basic model of a serious air traffic incident 344/07

4.3 *Model of air traffic accident*

Analysis of the factors leading to the incident may give an answer to the question what is the probability of such incident. In this case, such an analysis can be very interesting. It is, for example, to check how the situation would change if it was B767 the first aircraft to obtain permission to line up the runway. In this case the crew of the B737 would not have a reason to accept a permission to start issued for the B767.

In the presented example, however, a goal is to find a probabilistic dependence between the serious incident and an accident that could result from it. In this case, it is necessary to notice that it is sufficient that there exists only one additional factor, and incident would in fact be an accident. There are several scenarios that lead to an accident.

1 B767 crew, busy with their own take-off procedure does not pay attention to the message transmitted by radio by the ATR 72 pilot.
2 B767 crew takes a wrong decision to continue the take-off, despite noting B737 aircraft. Such a decision could arise, for example, with this reasoning: "there is no possibility to stop before the intersection, let B737 stop - after all, we have a permission to start, maybe we can pass the intersection before the B737", etc.
3 ATR 72 pilot does not watch the situation on the runways, just waiting for permission to line-up the runway.
4 ATR 72 pilot observes a dangerous situation, but does not immediately inform about it on the radio, instead discusses it with other members of his own crew.
5 Assistant controller does not pay attention to the information given by radio by the ATR 72 pilot, or does not respond to it properly - does not inform the controller.
6 Weather conditions (visibility) are so bad that it is impossible to see the actual traffic situation. This applies to B767, ATR 72 crews, and the air traffic controller.

All these scenarios will lead with certainty (or with great probability) to transformation of the incident into an accident, and can be analyzed using Petri net model. In this analysis we should take into account the possibility of occurrence of each scenario separately, as well as several of them at once.

4.4 *Probability of incident-accident transformation*

Analysis of the probability of transformation of incident into an accident must take into account the probability of each scenario mentioned above. Designation of some of these probabilities is very difficult or even impossible, because of the lack of statistical data, or it is even not possible to measure some values. In the case of scenario 6 we can use statistical data on meteorological conditions (visibility) in the airport. But in other scenarios, it is necessary to refer to experts' evaluation.

Taking into account the objectives of the analysis, it is possible to eliminate certain nodes without loss of accuracy, while simplifying the analyzed model. This applies, for example, to almost all the places and transitions associated with the process of taxiing and lining up the runway. For example, change of the set of places is determined as follows.

$$P_w = (P - P_r) \cup P_d \quad (9)$$

where: P_w - a set of places in the modelled accident,
P_r - a set of reduced places,
P_d - a set of places added to the model, to reflect the above-mentioned scenarios.

In this case (Figure 1) P_r = {"B767 awaiting permission to start", "B767 can line up RWY 33", "B767 on the RWY 33 threshold", "B767 ready for take-off", "B737 awaiting permission to start", "B737 can line up RWY 29", "B737 on the RWY 29 threshold", "B737 ready for take-off", "ATC not busy", "ATC busy", "ATR observes a simultaneous start"}.

On the other hand P_d = {„ATR warns?", "B737 continues to start", "B737 at the crossing", "B767 hears the warning?", "B767 continues to start", "B767 at the crossing", "B767 interrupts start?", "B767 begins deceleration", "weather?", "good visibility}.

A similar modification was made in regard to transitions, input, output and inhibition functions.

An additional issue to consider is change of transition type – from timed to immediate or vice versa. Petri net to model the transformation of the incident into accident, after reduction is shown in Figure 2. This network may be treated as a stochastic, timed, coloured Petri net. Its analysis allows observing some interesting relationships between a serious incident and the air traffic accident. It also allows determining some quantitative dependencies.

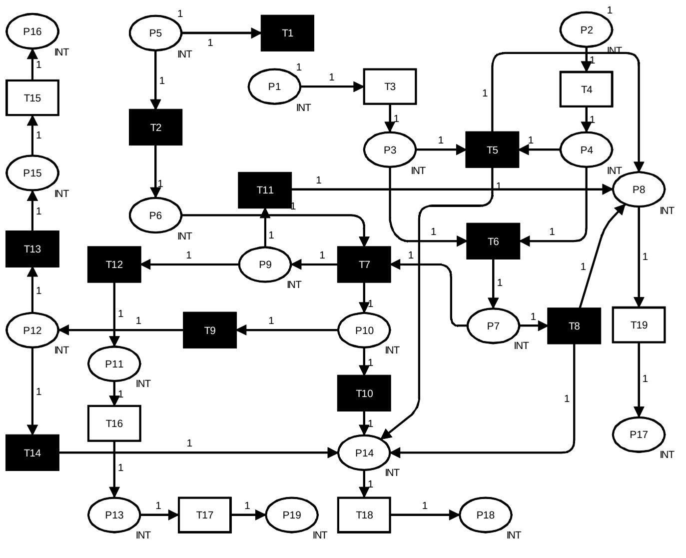

Figure 2. Model of serious incident 344/07 transformation into air traffic accident (after reduction of the states).

Assume the following places designations: p_1 – „B767 ready for take-off", p_2 – „B737 ready for take-off", p_3 – „B767 accelerates", p_4 – „B737 accelerates", p_5 – „weather?", p_6 – „good visibility", p_7 – „ATR warns?", p_8 – „B737 continues take-off", p_9 – „FD accepts warning?", p_{10} – „B767 hears warning?", p_{11} – „ATC sees conflict", p_{12} – „B767 interrupts take-off?", p_{13} – „B737 accepts order to interrupt take-off", p_{14} – „B767 continues take-off", p_{15} – „B767 begins braking", p_{16} – „B767 stops", p_{17} – „B737 at crossing", p_{18} – „B767 at crossing", p_{19} – „B737 stops".

The set of all states, called a reachability set, for model of accident is presented in Table 1.

The most important markings, from the perspective of the analysis presented in this article, are given in Table 2. Other states as well irrelevant places – were omitted.

Table 1. The reachability set for the model of accident arising from incident 344/07

M_0	$p_1+p_2+p_5$	M_1	p_1+p_2	M_2	$p_1+p_2+p_6$
M_3	p_2+p_3	M_4	p_1+p_4	M_5	$p_2+p_3+p_6$
M_6	p_4+p_6	M_7	p_3+p_4	M_8	$p_3+p_4+p_6$
M_9	p_8+p_{14}	M_{10}	p_7	M_{11}	$p_6+p_8+p_{14}$
M_{12}	p_6+p_7	M_{13}	p_8+p_{18}	M_{14}	$p_{14}+p_{17}$
M_{15}	p_8+p_{18}	M_{16}	$p_6+p_{14}+p_{17}$	M_{17}	p_9+p_{10}
M_{18}	$p_{17}+p_{18}$	M_{19}	$p_6+p_{17}+p_{18}$	M_{20}	$p_{10}+p_{11}$
M_{21}	p_8+p_{10}	M_{22}	p_9+p_{12}	M_{23}	p_9+p_{14}
M_{24}	$p_{11}+p_{12}$	M_{25}	$p_{11}+p_{14}$	M_{26}	p_8+p_{12}
M_{27}	p_9+p_{15}	M_{28}	$p_{11}+p_{15}$	M_{29}	$p_{11}+p_{18}$
M_{30}	$p_{13}+p_{14}$	M_{31}	p_8+p_{15}	M_{32}	$p_{11}+p_{16}$
M_{33}	$p_{13}+p_{15}$	M_{34}	$p_{13}+p_{18}$	M_{35}	$p_{14}+p_{19}$
M_{36}	p_8+p_{16}	M_{37}	$p_{15}+p_{17}$	M_{38}	$p_{13}+p_{16}$
M_{39}	$p_{15}+p_{19}$	M_{40}	$p_{18}+p_{19}$	M_{41}	$p_{16}+p_{17}$
M_{42}	$p_{16}+p_{19}$				

Table 2. Selected states of the system (model of accident)

	M_{18}	M_{19}	M_{40}	M_{41}	M_{42}
p_6 – good visibility	0	1	0	0	0
p_{16} - B767 stops	0	0	0	1	1
p_{17} - B737 at crossing	1	1	0	1	0
p_{18} - B767 at crossing	1	1	1	0	0
p_{19} - B737 stops	0	0	1	0	1

States M_{40}, M_{41}, M_{42} (called safe states) illustrate situations in which there is no accident. States M_{18} and M_{19} represent the situation where analysed incident transforms into accident. The joint probability of finding system in one of these states is the searched probability of incident-accident transformation. It can be determined both analytically and by simulation using a suitable software tool. In the present study a CPN Tools v. 3.0 package was used.

Analytical method for determining the sought probabilities will be presented on the example of the final state M_{19}. Partial subgraph of the reachability graph, for reaching M_{19} from initial state M_0 is shown in Figure 3.

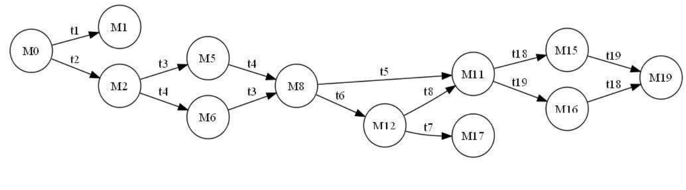

Figure 3. Partial subgraph of reachability of final state M_{19}.

Let's assume the following transitions designations: t_1 – „bad weather", t_2 – „good weather", t_3 – „B767 take-off phase I", t_4 – „B737 take-off phase I", t_5 – „ATR not watches", t_6 – „ATR watches", t_7 – „ATR warns", t_8 – „ATR not warns", t_9 – „B767 hears", t_{10} – „B767 not hears", t_{11} – „FD not accepts", t_{12} – „FD accepts", t_{13} – „B767 interrupts", t_{14} – „B767 not interrupts", t_{15} – „B767 decelerates", t_{16} – „ATC orders B737 to interrupt", t_{17} – „B737 interrupts take-off and stops", t_{18} – „B767 take-off phase II", t_{19} – „B737 take-off phase II".

Immediate transitions t_1, t_2, t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}, t_{13}, t_{14} are assigned weights, respectively: α_1, α_2, α_5, α_6, α_7, α_8, α_9, α_{10}, α_{11}, α_{12}, α_{13}, α_{14}. These weights are used to determine the probability of firing transitions in a situation of a conflict. Timed transitions t_3, t_4, t_{15}, t_{16}, t_{17}, t_{18}, t_{19} are assigned the intensities of realisation, respectively: μ_3, μ_4, μ_{15}, μ_{16}, μ_{17}, μ_{18}, μ_{19}. Also for this type of transitions in the event of a conflict, it is necessary to determine the probability of firing one of the conflicting transitions. This is done using the above intensities.

Because of the purpose of analysis, it is possible to reduce the reachability graph. Reduction consists of the removal of states that do not affect the probability of finding the system in the state M_{19}. Reachability graph after reduction is shown in Figure 4.

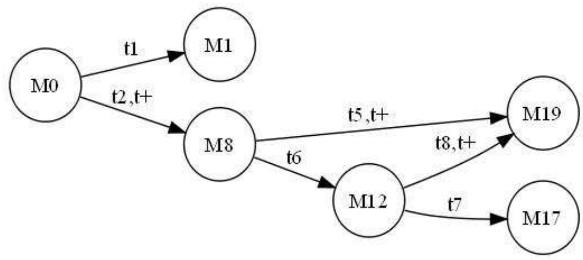

Figure 4. Reduced reachability graph for state M_{19}.

In this figure, the presence of an arc between nodes does not mean that there is a direct transition between states indicated.

In this case, the probability that the system will move from the state M_0 to M_{19} depends on the probabilities of firing of immediate transitions t_2, t_5, t_6 and t_8, and is described by the two sequences σ_1 and σ_2, and after reduction of intermediate states is as follows:

$$\sigma_1 = M_0[t_2, t_+\rangle M_8[t_5, t_+\rangle M_{19} \quad (10)$$

$$\sigma_2 = M_0[t_2, t_+\rangle M_8[t_6\rangle M_{12}[t_8, t_+\rangle M_{19} \quad (11)$$

$$P(M_0[\sigma_{1-2}\rangle M_{19}) = \frac{\alpha_2}{\alpha_1+\alpha_2} \cdot \left(\frac{\alpha_5}{\alpha_5+\alpha_6} + \frac{\alpha_6}{\alpha_5+\alpha_6} \cdot \frac{\alpha_8}{\alpha_7+\alpha_8}\right) \quad (12)$$

It is worth noting that in this case the probability of transforming incident into accident is not affected by intensities of timed transitions, but only the weights of immediate transitions.

5 SUMMARY AND CONCLUSIONS

In the paper the method of analysis of the relationship between the air traffic serious incident and accident was presented. The starting point for this analysis was the assumption that a serious incident describes a situation in air traffic, in which only one additional adverse event is sufficient to cause an accident. In the analyzed example (real air traffic incident), there are six scenarios, which lead to the transformation of an incident into accident.

Petri nets are used for modelling. Type of net used, depends on individual case and objective of analysis. In cases where the searched probability depends solely on events of the type of logical conditions (as in this example), or only on events that are characterized by the time - it is preferable to use the analytical variant. In cases when both types of events have an important role in transforming the incident into an accident - simulation variant is more efficient.

Coloured timed Petri nets are an important and convenient tool for analysis of traffic processes in air transport. Research performed has shown its usefulness in analyzing traffic safety problems. They are also efficient modelling tool in other modes of transport.

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