Model of air traffic in terminal area for ATFM safety analysis

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ABSTRACT: Air transportation system is understood as a set of elements (pilot, aircraft, air traffic control, ground ATC devices etc.), that are related to each other. Those relations are examined to check system’s possibility to fulfil air transportation tasks. The safety of the vehicles involved in the transport process is one of the most important criteria for its assessment. There are many factors which contribute to that safety, including technical reliability of the means of conveyance, traffic organisation, qualifications and abilities of people who operate the vehicles and supervise the transport process, etc. The main problem Air Traffic Services encounter during realisation of their tasks is excessive congestion of aeroplanes in the airspace. This congestion is especially well visible in terminal areas. In the paper the role of Air Traffic Flow Management (ATFM) unit in unloading this congestion and in air traffic safety was shown. Then the main steps of the method for determining airport capacity were examined. The method was used not only for calculating current airport capacity, but also for preparing some prognoses of future capacities of the airport. Probabilistic analysis of these prognoses was given. On this base some propositions of methods (algorithms) of co-operation between ATC and ATFM services were proposed.

1 INTRODUCTION

Air traffic is a large, dynamic, complex hierarchic system. Traffic control processes are of discrete nature. This control is carried out by ATC services. The main problem they encounter during realisation of their tasks is excessive congestion of aeroplanes in the airspace. This congestion is especially well visible in terminal areas. There were special ATFM (Air Traffic Flow Management) services established to unload this congestion. Traffic safety is one of their main goals, so it is necessary to develop methods of safety analysis of ATFM work.

The basic information that is necessary to fulfil this task is the airport capacity. To compute airport capacity, mathematical model of air space surrounding airport was elaborated. It is multi-phase, multi-line, queuing model. Because the processes of reporting and services occurring in this area make it impossible to determine analytically the characteristics of the system, a simulating computer model has been created. It makes possible to determine the capacity of airport for a given set of external conditions. Thanks to the applied method it makes also possible to determine in real time, prognoses of future capacities of the airport for chosen discrete moments in time.

Those prognoses are subject to some uncertainty. It is caused by the fact that the capacity of airport is unusually sensitive to even small changes of such difficult to predict factors as: the conditions of visibility, strength and direction of wind etc. In this paper the probabilistic analysis of the prognoses obtained by modelling has been made, assuming that both the changes of factors influencing on the airport capacity and the process of determining thereof are of discrete character. A discrete random variable $D_t$ was defined; which defines the capacity of airport at a certain moment $t$ in the future. The expected value of such random variable has been determined for: constant, known capacity decrease, as well as for the capacity decrease, whose value is defined by a discrete probability distribution. Changes of capacity lasting for one time interval have been analysed, as well as changes for several time intervals.

Detailed rules of air traffic services and ATFM services co-operation have been not precisely settled yet. Thus it is not determined when the information of anticipated airport capacity change should be passed to ATFM unit. On the one hand it should be done as fast as possible - so as to preserve much time to analyse and provide control process, leading to unloading traffic congestion. On the other hand
early prognosis is subject to relatively big error, and decisions undertaken on its basis may be not optimal. In both cases, mistakes may lead to decrease of the level of aircraft safety.

The discrete multicriterial programming problem was formulated on the assumption that expected values of airport capacity in discrete future moments in time are known. It leads to determination of the proper moment of passing the information of airport capacity change to ATFM services. It also helps in increasing the level of safety.

2 AIR TRAFFIC FLOW MANAGEMENT

The problem of determining the air traffic sectors capacity is a part of a wider problem called: Air Traffic Flow Management (ATFM). This problem consists in planning and controlling the streams of aeroplanes in the area of more than regional size in a long and a medium-term horizon of control.

The services dealing with planning of the flow of streams analyse the capacity of individual control sectors and accordingly to the results of this analysis suitably co-ordinate formation of these streams. The aim of these services is to minimise the waiting times of aircrafts being in the air, to avoid temporary overload in individual sectors, to increase the regularity of air traffic.

ATFM Services realise these tasks at many levels and with the use of varied technical and organisational means. Acting strategically they analyse and co-ordinate the flight plans so as to minimalize the probability of occurrence of overload. Tactical operations consist in suitable delaying of take-offs of aircrafts, which would find themselves in overloaded sectors of control (Richetta et al. 1994). The basis for these operations is the fact that delays of aircrafts realised on the ground surface involve smaller costs than those resulting from waiting in the air for the possibility to land. In the latter case additional costs are costs of fuel and those connected with safety of passengers.

Independently from the type of realised ATFM tasks, the service responsible for them must know current and future (foreseen) capacity of control sectors, especially in terminal area. This fact is direct cause of undertaking works aiming at establishing methods, standards or even guidelines for determining the capacity of sectors of air traffic.

To have these done effectively by ATFM services it is necessary to inform them about changes in capacity of control sectors with the time horizon not larger than one hour. For the services controlling air traffic it seems necessary to have suitable technical devices, which will be able to determine the capacity in real-time.

3 METHOD OF DETERMINING THE AIRPORT CAPACITY

It has been proposed to carry out a study over a terminal area seen as a network, multi-phase queuing system, in which the output streams from one sub-system (phase) are the input streams for the next sub-system (the next phase of service). If in the case of an analytic model we have to deal with a kind of very complex queuing model, for which determining stationary characteristics is at present impossible, in the case of a simulating model the numerical experiments show its big simplicity and easiness of obtaining the results.

The proposed method of solution of the problem of airport capacity consists of the following stages:

1. Determination of model of studied space.
2. Recording the model with the use of special language of description.
3. Execution of series of simulating experiments
4. Analysis of received statistical sample and determining the capacity.

A detailed description of the method can be found in (Skorupski 2002).

3.1 Model of studied space

The method of calculation the capacity of airport enables to determine the capacity of any airport. In order to do that, on the basis of the worked out methodology one should perform complete identification of phenomena in the studied system. These are, amongst others:

- The character of stream of reports to the TMA area (or the CTR, if the TMA region in the studied case has not been delimited): the distribution of probability of reports, number of aircrafts of individual weight categories which occur in the stream of reports.
- The number and configuration of the airways in the TMA region: the distance from the border of TMA to the border of CTR, class and technical equipment of these airways.
- The configuration of runways: their number, distance between runways and mutual location.
- Weather, particularly visibility, because this has large influence on applied principles of air traffic.
- The strength and direction of wind: it determines then possibility of using some runways; it has also influence on the way the stream of aircrafts going out from the TMA space.
- State of surface of runways and taxiways.
- The limitation connected with legal regulations concerning noise: they have influence on possibility of using some areas of space of the region of airport by some types of aircrafts for some purposes.
The operational strategy of controlling applied by the controllers: for example using some runways only for landings or only for take-offs.

- The equipment of services of air traffic control with navigational devices of type ILS, VOR, DME and radar devices.

3.2 Determining the capacity

As the result of executing the series of simulating experiments a random sample of realisation of two-dimensional random variable \((X,D)\) is obtained. Pairs \(X,D\) define: intensity of air traffic, size of average delay.

On the basis of these data a table is created, which corresponds to the distribution of probability of relative average delay at the condition \(X=x_0\) for all accessible values \(x_0\) (intensity of air traffic). For this table the values of function \(G_\lambda\) are calculated, which is the probability of that the average delay is larger or equal to a pre-set maximum value \(D_{\max}\). The probabilities are determined for all accessible values of \(x_0\).

For function \(G_\lambda\) the value of inverse function is determined in point \(\frac{1}{\lambda}\). Because the inverse function can be not unique (and it usually is not) the smallest and the largest value fulfilling the pre-set condition is read, and from these values arithmetic average is then calculated.

4 USE OF THE SYSTEM TO PROGNOSE THE AIRPORT CAPACITY

The main objective, for which in this work it has been postulated to create the system of determining the capacity of airport, which will give results in real time, is making possible to calculate the capacity of airport in future moments. These calculations would be possible to be done after determining the capacity of airport at the current moment.

In order to apply the existing system for determining the future value of capacity of airport one should do the following:

1. estimate the future values of input parameters,
2. update input files with the use of the program of modification of parameters
3. carry out simulating experiments,
4. determine the future capacity.

One should be aware that this kind of calculations has the character of prognoses. This means that they are burdened with some error. The source of this error is the uncertainty as to the verity of the premise inducing us to assume that the capacity of airport will change. Such a premise can be e.g. a meteorological prognosis about a storm coming into the TMA region. Even accepting that the method of determining the capacity brings as effect true results (being in accordance with real capacity), still there exists some uncertainty, whether this kind of premise defines correctly the time, the intensity, and the place of occurrence of the phenomenon influencing on the change of capacity.

The problem of uncertainty as to the result of the prognoses of future airport capacity is not only a theoretical problem. The air traffic control services are obliged to inform the ATFM organs about foreseen changes of capacity. Therefore there arises a question related to this: whether, and in which moment to inform ATFM services about the foreseen change of capacity. On the one hand this should be done as early as possible - in order to get as much time as possible for analysis and realisation of process of unloading the overload in the traffic. On the other hand, however, such an early prognosis is burdened with comparatively large error. So there comes up a presumption that it is better to wait, not informing the ATFM services, until the phenomenon which influences on the change of capacity has been better recognised. Thus we can with a higher probability confirm or reject the occurrence of the premise to change the capacity. Such action has, however, also some unfavourable result - if the premise is confirmed, the ATFM organs will have less time for taking remedial means. When the information comes too late taking any remedial steps can turn out quite impossible.

5 PROBABILISTIC ANALYSIS OF OBTAINED PROGNOSES

In relationship with the fact, that the calculations of future capacity of region of airport are burdened with uncertainty we can define a random variable \(D_t\), defining capacity of airport at certain moment \(t\) in the future. Moreover, there should be rather used the term: expected value of future capacity. The random variable \(D_t\) is dependent on the occurrence of phenomena causing the change of capacity.

Unfortunately, some of these phenomena also have probabilistic character and we can only talk about the occurrence of premises suggesting the change of capacity. It is possible for these premises to try determining the probability of occurrence of the phenomenon to which the given premise testifies. It is also possible to define the value of intensity of the discussed phenomenon.

It is assumed that the problem is considered for one day, it means the following moments are considered:

\[ j: j=t_1,t_2,...,t_K \]  

(1)

where \(t_1\) - the moment of beginning of traffic in a given airport, \(t_K\) - the moment of end of traffic.

For the majority of airports there occurs a natural night pause in activity. It results both from demand on flights (minimum during the night) as well as
5.1 Constant value of change of capacity, time of duration of change - 1 interval

It is assumed, that in moment \( k \) there becomes recognised the premise on the basis of which the prognosis is formulated, that in moment \( x \) the change of capacity of the region airport will happen. Let’s put that the change of capacity will depend on its lowering by a constant value \( \delta C \). Moreover, this change will last for one time interval \( x \) only. The probability, that the change prognosed at the moment \( k \), will really happen at the moment \( x \) is equal to \( P_k^x \).

The probabilities of capacity at the moment \( x \) are then equal to

\[
P(D_x = C_T - \delta C) = P_k^x
\]

(2)

\[
P(D_x = C_T) = 1-P_k^x
\]

(3)

The expected value of capacity of the region airport at the moment \( x \), defined at the moment \( k \) is therefore

\[
E(D_x) = (C_T - \delta C) \cdot P_k^x + C_T \cdot (1 - P_k^x)
\]

(4)

\[
= C_T - P_k^x \cdot \delta C
\]

5.2 Variable value of change of capacity, time of duration of change - 1 interval

It is assumed, that in moment \( k \) there becomes recognised the premise, authorising to presumption that at the moment \( x \) lowering of capacity will happen. The extent of this lowering is a random variable, which takes values \( \delta C = \delta_0, \delta_1, ..., \delta_{DM} \). On the set of value \( \delta C \) it is possible to determine the distribution of probability of their occurrence.

\[
R_{\delta_C} = \sum_{i=0}^{DM} R_{\delta_i} = 1
\]

(5)

Therefore the expected value of the lowering of capacity will be

\[
E(\delta C) = \sum_{i=0}^{DM} \left( R_{\delta_i} \cdot \delta_i \right)
\]

(6)

The formula for the expected value of capacity at the moment \( x \), prognosed at the moment \( k \) can therefore be modified to the form:

\[
E(D_x) = C_T - P_k^x \cdot E(\delta C)
\]

\[
= C_T - P_k^x \cdot \sum_{i=0}^{DM} \left( R_{\delta_i} \cdot \delta_i \right)
\]

(7)

5.3 Constant value of change of capacity, time of duration of change - several intervals

In the case of taking the length of time interval equal 1 hour it is possible, that the change in capacity will last for one time interval. However, both in this case, and in the case of shorter time intervals it is extremely probable that the change in capacity will keep on for at least several time intervals.

Let’s accept therefore that at the moment \( k \) the occurrence of the premise has been affirmed, on basis of which it is possible to formulate the prognosis that in time intervals \( x_0, x_1, ..., x_L \) there will occur lowering of the capacity by a constant value \( \delta C \).

Then

\[
\forall x_j \in [x_0, x_1, ..., x_L] \quad E(D_{x_j}) = C_T - P_k^{x_j} \cdot \delta_C \quad (8)
\]

If the probabilities of occurrence of assumed constant reduction in capacity are equal in each time intervals, then obviously \( P_k^{x_0} = P_k^{x_1} = ... = P_k^{x_L} = P_k^x \).

One can assume, however, that these probabilities are different, in dependence on the number of time intervals. For example the smallest in time intervals \( x_0 \) and \( x_L \), whereas the largest in the centre of the time interval \([x_0, x_L] \).

5.4 Variable value of change of capacity (the same distribution), time of duration of change - several time intervals

Here it is assumed, that for each time interval \([x_0, x_L]\) there is proposed the possibility of occurrence of lowering of capacity of different intensity. It is a random variable, taking values \( \delta C = \delta_0, \delta_1, ..., \delta_{DM} \). It is also assumed that for every time interval \( x_j \) for the set of values \( \delta C \) the identical distribution of probability can be determined.

\[
\forall x_j \in [x_0, x_L] \quad R_{\delta_C} = \sum_{i=0}^{DM} R_{\delta_i} = 1
\]

(9)
Therefore the expected value of lowering of capacity will be the same for each section of the considered time interval \([x_0,x_L]\) and it will be

\[
E(\delta_c) = \sum_{i=0}^{DM} \{ R_{\delta_i} \cdot \delta_i \}
\]

(10)

The formula for expected value of capacity at the moments \(x_j\), prognosed at the moment \(k\) can be therefore modified to the form:

\[
\forall x_j \in \{ x_0, x_1, \ldots, x_L \}
\]

\[
E(D_{x_j}) = C_T - P_{x_j} \cdot E(\delta_c) = C_T - P_{x_j} \cdot \sum_{i=0}^{DM} \{ R_{\delta_i} \cdot \delta_i \}
\]

(11)

5.5 Variable value of change of capacity (any distribution, the time of duration of the change - several time intervals

Let’s now consider the case, when in each section of time interval of lowered capacity \([x_0,x_L]\) the intensity of reduction of capacity is given with a different distribution of probability. Then one should define the whole family of distributions of probability with the following form:

\[
R_{x_j}^{\delta} = \sum_{i=0}^{DM} R_{\delta_i} = 1, \text{where } x_j \in \{ x_0, x_1, \ldots, x_L \}
\]

(12)

For each of considered sections \(x_j\) of lowered capacity the expected value of magnitude of reduction will be in this case

\[
E(\delta_{c,j}) = \sum_{i=0}^{DM} \{ R_{\delta_i} \cdot \delta_i \}
\]

(13)

Finally the expected value of capacity at the moments \(x_j \in \{ x_0, x_1, \ldots, x_j \}\), prognosed at the moment of occurrence of the premise \(k\) will be given by the following formula:

\[
E(D_{x_j}) = C_T - P_{x_j} \cdot E(\delta_{c,j}) = C_T - P_{x_j} \cdot \sum_{i=0}^{DM} \{ R_{\delta_i} \cdot \delta_i \}
\]

(14)

\[
x_j = x_0, x_1, \ldots, x_L
\]

6 CO-OPERATION WITH ATFM SERVICES

The detailed rules of co-operation of ATC services with the ATFM services have not been so far defined exactly. So it is also not defined when the information about a foreseen change in capacity should be passed on to the ATFM organ.

One should notice here the technology of work of the ATFM services. They haven’t been so far equipped with tools, algorithms, programs enabling to fulfil their tasks in effective way. Their operation is based on solutions worked out currently by a group of experienced controllers. On the basis of analysis of the existing and the future traffic situation they work out decisions for controlling the streams of aircrafts.

So, it is visible, that the prognoses of type ‘About hour ... the capacity will be ... with probability ...’ are with this technology of work almost completely useless. One should rather work out the prognoses in the form ‘About hour ... capacity will be exceeded by ...’. In order to do this it is necessary to carry out the optimisation of the time of communication of information about prognosed capacity, from the point of view of minimisation of possibility of making mistake and maximisation of time available for intervention for the ATFM services.

In order to do that it is possible to propose certain mathematical model. In this model there will be used observation that the assessment of premise inducing us to undertaking prognosis can evolve in the time. The following scenarios are possible:

1. The premise to change of capacity is entirely incorrect – the capacity will not change.
2. The premise is underestimated – the capacity will change to the foreseen extent, but earlier.
3. The premise is underestimated – the capacity will decrease, but to larger extent.
4. The premise is overestimated – the capacity will decrease, but later than prognosed.
5. The premise is overestimated – the capacity will decrease, but less than assumed.
6. The assessment of validity of premise is correct – the capacity will be in accordance with the prognosis.

Of course, there are possible intermediate cases (especially for points 2 and 3 as well as 4 and 5), but we will not consider these for the reason of simplifying.

Let’s put that the expected value of capacity \(E(D_{x_j})\) is given in each time interval in which we prognose it’s lowering.

For each of moments \(j\) we can determine the number of aircrafts \(N_j\) landing on the airport in the time interval \(j\).

Let’s define moreover \(N_{ij}\) as the number of aeroplanes, which start at the moment \(s\) and land on studied airport at the moment \(j\). Let’s notice, that

\[
N_j = \sum_{s=t_0}^{t_1} N_{ij}
\]

(15)

assuming, that all moments preceding \(t_1\) are marked as \(t_0\).

Let’s mark by \(M_{kj}\) the number of aeroplanes which can be stopped on the ground at the moment \(k\), i.e. the aeroplanes, which have not taken off by the moment \(k\), and whose planned arrival is to take place at the moment \(j\). We can see, that
\[ M_{ij} = \sum_{s=s_{ij}}^{x} N_{ij} \]

If \( N_{ij} - E(D_{ij}) \) is positive we have to deal with the case when one should limit the traffic because the capacity has been surpassed. In the opposite case it is possible to wait for a more detailed premise to induce us to prognose. Let\'s accept like previously, that premise has been observed at the moment \( k \) and on the basis thereof we prognose, that at the moment \( x \) lowering of capacity to value \( E(D_{ij}) \) will happen. Knowing, that the probability that observed phenomenon will cause occurrence of phenomenon reducing the capacity is equal \( P_{ij}^{s} \) we can formulate the following task of multi-criterion programming:

\[
\begin{align*}
\text{max} & \quad M_{xs} \\
\text{min} & \quad \varepsilon_{j}^{s} \\
\text{bounded} & : \\
N_{s} - E(D_{s}) & > 0, \\
N_{s}, M_{xs} & \geq 0 \text{ and } \text{int} \\
j & = k, k+1, ..., x
\end{align*}
\]

where \( \varepsilon_{j}^{s} \) is the measure of error made during prognosing at the moment \( j \) about the change of capacity at the moment \( x \).

This error is the largest in the case, when probability that the capacity at the moment \( x \) is \( D_{s} \), equals \( \frac{1}{2} \), whereas the smallest when this probability is equal 0 or 1. It is possible therefore to define it as follows:

\[
\varepsilon_{j}^{s} = 1 - \left| 1 - 2 \cdot P_{ij}^{s} \right| \tag{18}
\]

which means that

\[
\varepsilon_{j}^{s} = \begin{cases} 
2 \cdot P_{ij}^{s} & \text{for } P_{ij}^{s} \leq \frac{1}{2} \\
2(1 - P_{ij}^{s}) & \text{for } P_{ij}^{s} > \frac{1}{2}
\end{cases} \tag{19}
\]

This task can be, of course, reduced to a one-criterion problem, by transfer of the criterion of maximisation of the number of aeroplanes which can be delayed on the ground to the boundary conditions. It can be done as follows:

\[
\begin{align*}
\text{min} & \quad \varepsilon_{j}^{s} \\
\text{bounded} : \\
M_{xs} & \geq N_{s} - E(D_{s}) \\
N_{s} - E(D_{s}) & > 0, \\
N_{s}, M_{xs} & \geq 0 \text{ and int} \\
j & = k, k+1, ..., x
\end{align*}
\]

Obviously this second formulating can lead to solutions which give a lot fewer possibilities of intervention for the ATFM services. In the boundary situation this intervention would be stopping on the ground all aeroplanes landing on the studied airport at the moment \( x \), for which this is possible. Such solutions can not be accepted by the ATFM services.

The task is strongly dependent on time. Probability \( P_{ij}^{s} \) can undergo changes in time because factors taken into account in prognosis changes, confirming or denying the put forward thesis. Time in this problem is considered in the form of temporary moments \( j=k, k+1, ..., x \), so it is in discrete form.

This suggests the possibility of defining the task with the use of the methods of Dynamic Programming.

At the stage \( j \) (at time moment \( j \)) decide whether to inform the ATFM services about appearing of the lowering of capacity. This means that possible decisions are:

\[
x_{j} = \begin{cases} 
1 & \text{if ATFM has been notified} \\
0 & \text{if contrariwise}
\end{cases} \tag{21}
\]

The state of the system is equal to:

\[
s_{j} = M_{xs} - (N_{s} - E(D_{s})) \tag{22}
\]

which corresponds to the number of aeroplanes, by which the number of aeroplanes possible to be delayed on the ground is larger than the shortage of capacity of the target airport.

\[
f_{j}(s_{j}, x_{j}) - \text{the minimum error of prognosis at the moment } j \text{ about capacity at the moment } x, \text{ with the assumption, that the state of system is equal } s_{j}^{x}, \text{ and that at the time moment } j \text{ the decision } x_{j} \text{ about informing or non informing the ATFM services about change of capacity was taken.}
\]

\[
f_{j}(s_{j}) - \text{the minimum error of prognosis at the moment } j \text{ about capacity at the moment } x, \text{ with the assumption, that the state of system is equal } s_{j}^{x}.
\]

With so defined a problem of Dynamic Programming we can determine recurrent dependence:

\[
f_{j}(s_{j}, x_{j}) = \min \{ \varepsilon_{j}^{s} | x_{j} = 1 \text{ or } s_{j}^{s} \leq 0 \}
\]

\[
\text{if } x_{j} = 0 \text{ and } s_{j}^{s} > 0
\]

If we accept, that the value of variable of state for which we take the decision \( x_{j}=1 \) is not equal zero, but to certain minimum value set by the ATFM service, one should suitably modify the above mentioned recurrent dependence.

7 CONCLUSIONS

Air Traffic Flow Management services play a very important role in increasing air traffic safety. The aim of these services is to minimise the waiting times of aircraft being in the air, so to avoid temporary overload in sectors. ATFM must know current and future (foreseen) capacity of control sectors, es-
pecially in terminal area. It is necessary to inform them about changes in capacity of control sectors with time horizon not larger than one hour.

However the detailed rules of ATC services and ATFM services co-operation have been not precisely settled yet. Thus it is not determined when the information of anticipated airport capacity change should be passed to ATFM unit. On the one hand it should be done as fast as possible - so as to preserve much time to analyse and provide control process, leading to unloading traffic congestion. On the other hand early prognosis is subject to relatively big error, and decisions undertaken on its basis may be not optimal. That is why probabilistic analysis of future capacity prognoses should be performed in real-time. In both cases, mistakes may lead to decrease of the level of aircraft safety.

On the basis of above mentioned prognoses, the discrete multicriterial programming problem was formulated on the assumption that expected values of airport capacity in discrete future moments in time are known. It leads to determination of the proper moment of passing the information of airport capacity change to ATFM services. It also helps in increasing the level of safety.

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