Expert Systems with Applications 41 (2014) 7406–7414

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Multi-criteria group decision making under uncertainty with application to air traffic safety

Jacek Skorupski *

Warsaw University of Technology, Faculty of Transport, Warsaw, Poland

ARTICLE INFO

Keywords:
Group decisions and negotiations
Multiple criteria analysis
Fuzzy sets
Air traffic
Traffic safety

ABSTRACT

There are many methods for solving problems of multi-criteria group decision making under uncertainty conditions. It is quite often that decision makers cannot formulate unequivocally their individual preference relations between variants. Analysing the causes of a serious aircraft incident is an example where a group of experts is required to have a very detailed yet interdisciplinary knowledge. Obviously, each expert has only a fraction of such knowledge. Hence, experts can make fuzzy evaluations when they are not sure about them or it is not possible to gain full knowledge. There is a need for a method that in such a case takes into account the strength of preference expressed in the significance of each criterion. Both the significance of criteria and the scores assigned to variants can be represented using fuzzy expressions.

The proposed method reflects the problems of decision making when both objective (represented using non-fuzzy expressions) and subjective (represented using linguistic expressions) criteria, are involved. The proposed method enables to obtain a solution without having to conduct negotiations between decision makers. This is of advantage when there is a risk that some experts will be dominated by others. The method not only helps define a single preferred solution but also create the preference relation within a group. By applying this method, it is possible to reproduce the actual preference relations of individual decision makers. Presenting them to decision makers may induce them to change their evaluation of the weights of criteria or how they score variants.

1. Introduction

In the contemporary world many decision problems pose a major challenge for decision makers. On the one hand, the systemic approach prevails, in which as many effects of decisions made have to be taken into consideration as possible. This, in turn, causes that a large array of evaluation criteria have to be used to find the optimal decision. On the other hand, applying a great number of evaluation aspects requires a very extensive knowledge. At the same time, for the assessment of individual permissible decisions to be accurate, it has to be an in-depth knowledge. Under such conditions it may be impossible for a single decision maker to make responsible decisions. Then a group decision to be made by a number of decision makers or, as the case may be, a single-person's decision based on the opinions of many experts is sought (Kocher, Strauß, & Sutter, 2006).

However, as far as expert evaluations are concerned, it is generally known that they are very often descriptive and unclear. This is due to many factors, for example, unavailability of full knowledge of a phenomenon or difficulty in a precise and formal definition of relations or relationships. Therefore, the decision problem has to be addressed in the context of decision making under uncertainty (Dubois & Prade, 1992). In addition, evaluation criteria may be fuzzy, as may also be their significance to the selection of the optimal variant. All this may place a decision problem among considerations described using e.g. fuzzy or rough set theory (Greco, Matarazzo, & Slowinski, 2001).

This paper proposes a method for solving problems of multi-criteria group decision making under uncertainty conditions. The method represents a new approach based on the assumption that decision makers cannot formulate unequivocally their individual preference relations between variants, but at the same time takes into account the strength of preference (Hamouda, Kilgour, & Hipel, 2006). In addition, it presumes that negotiations between decision makers (experts) are impracticable or inadvisable and the decision has to be based on their one-off evaluations. The third important aspect of the approach is to include in the evaluation criteria of variants both objective criteria (represented using non-fuzzy expressions) and those that are subjective (represented

* Tel.: +48 22 234 7339.
E-mail address: jsk@wt.pw.edu.pl

http://dx.doi.org/10.1016/j.eswa.2014.06.030
0957-4174/

using linguistic expressions) (Merigó, Casanovas, & Yang, 2014; Rao & Patel, 2010). The weight of individual criteria is derived from the aggregated weights assigned to them by each decision maker using linguistic variables.

The presented method can be the basis for building a decision support system, which can draw conclusions and make decisions which consist in indicating a solution preferred by the group. This system uses knowledge base that comes from experts who express it in the form of multi-criteria ratings of variants and criteria weights. Such expert systems are very necessary in practice, especially when each expert has only partial competence in the problem studied. An example from the field of aviation safety, presented in Section 4, belongs to a class of problems for which there is no formal mathematical model of problem solving algorithm. Part of the knowledge, necessary to build the knowledge base for the expert system, is available and expressed precisely, but some knowledge is uncertain and subjective. Hence, within the proposed model for inference engine, both objective and subjective criteria for evaluating the alternatives appear.

The paper is organised as follows: Section 2 briefly discusses the problem of multi-criteria group decision making under uncertainty. This section also provides a short overview of the literature. Section 3 provides a description of the new proposed approach to solving the problem of multi-criteria group decision making under uncertainty conditions. Section 4 presents an example of applying the developed approach. It has been used to illustrate the problem of attempting to rank in significance the causes of the serious aircraft incident that actually occurred at the F Chopin Airport in Warsaw. This ranking is the final step in the work of the aircraft accident investigation team, which is composed of experts in various fields. This issue is extremely important to improving air traffic safety because for major causes of aircraft incidents recommendations are issued that specify what actions are to be taken in the future. The paper is closed with the summary and conclusions contained in Section 5.

2. Multi-criteria group decision making under uncertainty

2.1. Group decision making

Group decision making has been discussed in the world's literature for many years. Obtaining a mutual decision agreed on between all decision makers is a major difficulty here. Decision makers are often individuals of strong personality, who are able to persuade others to accept their argumentation and judgement even if in reality such argumentation and judgement are not correct. Similarly, persons of weaker personality will not be successful with their standpoint despite it being sound (de Wit, Jehn, & Scheepers, 2013). The problem becomes more complicated when it is a multiple-criteria evaluation and each decision maker has a different hierarchy of criteria significance. The literature often refers to the need to reach a consensus (Wibowo & Deng, 2013; Zhang, Zhang, Lai, & Lu, 2009). This may be a long-lasting process, however, which does not necessarily have to lead to finding the best solution. The main reasons are stated above. Another one may be the need to keep the assessments of individual experts anonymous. An example of such an example is the evaluation of personnel, where it is necessary to exclude psychic pressure when making judgements. This issue is discussed in (Yu, Zhang, & Xu, 2013). Xiong, Tan, Yang, and Chen (2013), in turn, proposes a model of supporting group decisions in which a group decision is evaluated using two measures. One measure is the acceptance of a reached decision among group members. The other one is the vulnerability of a solution to changes in the preferences of individual decision makers. An interesting method of developing a shared standpoint while minimising necessary concessions is presented in (Zhang & Dong, 2013).

This paper focuses more on making decisions through the evaluation of variants by a group of experts, whose primary objective is rather delivering a reliable opinion, which e.g. enables to create a rank list of various solutions, than working out a common decision. There are many examples of such problem-solving strategies, e.g. a review of research grant applications based on a number of reviewer's views (Cook, Golany, Penn, & Raviv, 2007). Group decision-making is also used for transport-related purposes. For instance, (Rosmuller & Beroggi, 2004) discusses how group decision making methods can be used in the design of railway infrastructure, with particular attention paid to safety criteria. (Tavana, Khalili-Damghani, & Abtahi, 2013) presents a method of prioritising advanced technological projects at NASA, whereas (Chuu, 2011) discusses a model of group decision making for flexible supply chain management using a fuzzy linguistic approach. And (Yousefi & Hadi-Vencheh, 2010) presents how group decision making methods can be used to chart development directions for the automotive industry.

Section 4 presents an example solution to such a problem with regard to air transport, and more precisely in analysing the causes of a serious aircraft incident.

2.2. Multi-criteria analysis

The approaches to group decision making discussed in Section 2.1, whether they address the issue in terms of fuzziness or not, are based on the assumption that decision makers know how to describe the preference relations for a set of alternatives. For the ith decision maker such a relation can be defined as non-fuzzy preference order

$$O_i \subset A \times A \tag{1}$$

which means that if the ith decision maker prefers variant a_1 to a_2, then $(a_1,a_2) \in O_i$. It can also be written as

$$O_i = (\cdots \succ a_1 \succ a_2 \succ \cdots) \tag{2}$$

A reasonable preference relation should be transitive, reflexive and asymmetric. There are many solutions leading to accepting a reasonable order of variants for a whole group of experts based on individual preference relations. Among the most interesting papers are those by Zhang, Dong, and Xu (2012) and Kacprzyk (1986). The first proposes a model for incomplete additive preference relation that aims to calculate a complete fuzzy preference relation. The latter presents a method leading to a solution in terms of fuzzy values, although it does not include decision making uncertainty. These two methods share one major drawback, however. They do not take into account the strength of the decision maker's preferences. It is identical for them whether the decision maker prefers variant a_1 to a_2 to a minimal degree (the decision makers finds them almost indifferent) or does so very strongly. The approach presented in this paper eliminates that drawback by enabling decision makers to define and including the strength of preferences expressed using evaluation criteria and weights. A somewhat similar approach is mentioned in Chin and Fu (2014) and Wu and Chiclana (2012).

In multi-criteria evaluation of variants, the assumed existence of a transitive, reflexive and asymmetric preference relation does not have to be true. Decision makers often formulate preference relations that are e.g. non-transitive. It is true that they change their mind and modify the preference order of alternatives when they realise it but then doubts may arise whether the finally formulated preference relation reflects the decision maker's actual opinion (Tsai & Böckenholt, 2006).

There are numerous non-fuzzy methods of evaluating variants under multi-criteria conditions. An overview of them can be found e.g. in (Cho, 2003). Most often various ways of aggregating criteria into a single metacriterion (Stewart, French, & Rios, 2013) are used in them. Another typical approach is to employ an interactive procedure, in which further better solutions are found as the decision makers expresses his or her preference for pairs of alternatives (Chaudhuri & Deb, 2010). Another approach is to find solutions that are most similar to the ideal solution or that which is most distant from the anti-ideal (Dheena & Mohanraj, 2011).

This paper approaches multi-criteria evaluation of variants when there are many decision makers, with both objective and subjective criteria and fuzzy criteria strength.

2.3. Uncertainty in decision making

Many contemporary systems and processes are so complex that interdisciplinary yet very detailed and specialist knowledge is required to evaluate them. When opinions of many experts are used, these are most often specialists in their precise fields and knowledge available to each expert is different. In some approaches rates of the expert's importance are used in such a case which reflect the expert's contribution to reaching a group opinion (Guha & Chakraborty, 2011; Yue, 2012). Such a solution has a major drawback, however. In fact, by diminishing the role of some experts, we lose the advantage of group decision making, as it becomes one-person decision making by the decision maker with the highest rank. Perez, Cabrerizo, Alonso, and Herrera-Viedma (2014) attempt to eliminate this drawback by including the weights of opinions expressed by individual experts not only in aggregations of their preferences but also in the process of supporting experts in changing their preferences. However, the problem of arbitrarily assigning expert weights and thus eliminating (diminishing the significance of) some opinions is still present.

The lack of full expert knowledge about all aspects of a particular issue is a major source of uncertainty for the problem in question. When an expert's knowledge of a problem is incomplete, he or she cannot assess it in an entirely correct way. Nor can he or she fully determine which criteria are to be considered as most important. Hence, those group decision making methods which are based on reaching agreement are inflicted with the drawback of forcing experts to express their opinions on fields (include in their evaluations fields) in which they are not fully competent (Zgurovsky, Totsenko, & Tsyganok, 2004).

The method proposed in this paper accepts another approach. Each expert freely formulates his or her opinion whatever knowledge he or she has. He or she is not forced to agree his or her opinion with another expert whose knowledge may substantially differ (be more or less extensive) in a specific area. But he or she can freely assign not only scores to alternatives but also weights to individual evaluation criteria. Thus he or she can fully express his or her opinion on a variant under evaluation, making maximum use of his or her knowledge in the field in which he or she is most competent. At the same time, by using linguistic evaluations, he or she can express his or her opinions less categorically where his or her competence is lower than that of another decision maker.

3. Method of multi-criteria group decision making under uncertainty conditions

The decision is made by a group of decision makers

$$D = \{d_i\}, \quad i = 1, \dots, d \tag{3}$$

The decision makers' task is to choose the best variant from a set

$$A = \{a_j\}, \quad j = 1, \dots, a \tag{4}$$

The decision makers assess each variant using an adopted set of criteria. The criteria can be objective, e.g. the acquisition cost of equipment representing variant a_j, the number of people needed to operate it, the distance between the start and end points (if the variant consists in selecting an itinerary), etc. In such a case the decision maker's problem lies only in giving a score to the evaluation of a variant according to a predefined criterion.

Another type is subjective criteria such as travel comfort, solution appearance or its prestige. Such criteria often determine which solution is chosen. Subjective criteria have to be expressed using subjective scales. They are usually linear intervals or descriptive scales. This paper accepts that assessments can be given by assigning to variants a linguistic variable equivalent to a fuzzy set with a specific characteristic function.

Let us denote the set of objective criteria by

$$CO = \{c_k\}, \quad k = 1, \dots, co \tag{5}$$

and the set of subjective criteria by

$$CS = \{c_k\}, \quad k = co + 1, \dots, co + cs \tag{6}$$

where co and cs are the number of objective and subjective criteria, respectively.

Thus we can define two types of evaluation functions: objective

$$vo: \ D \times A \times CO \rightarrow \mathbb{R} \tag{7}$$

and subjective

$$vs: \ D \times A \times CS \rightarrow B \tag{8}$$

where: $vo(d_i,a_j,c_k), c_k \in CO$ – stands for the numerical score given by the ith decision maker to the jth variant (solution) according to the kth (objective) criterion, $vs(d_i,a_j,c_k), c_k \in CS$ – stands for the fuzzy linguistic score given by the ith decision maker to the jth variant (solution) according to the kth (subjective) criterion, $B = \{(x,\mu_B(x)): x \in X,\ \mu_B(x) \in [0,1]\}$ – stands for the fuzzy set with membership function μ_B, representing the fuzzy linguistic score.

From a single decision maker's point of view, the final evaluation of variants is a linear combination of the score and the weight of a criterion. With regard to the classic tasks of group decision making, it is often very difficult for decision makers to assign weights to individual criteria, especially when there is a large number of criteria or some criteria are indifferent to a decision maker. Even if they make it, e.g. using pair comparisons, such assignments are often contradictive to each other or fail to meet basic requirements, e.g. a preference relation is non-transitive. There are methods to find the actual preferences based on the choices made, e.g. (Rose & Hensher, 2004). They are inaccurate and unreliable, however. A markedly better mapping of the decision maker's actual preferences in relation to criteria can we obtained with inaccurate evaluations using linguistic variables, such as *'important'*, *'of little importance'*, etc. Let us assume then that the weights of individual criteria are fuzzy values

$$w : D \times (CO \cup CS) \rightarrow K \tag{9}$$

where

$$K = \{(y, \mu_K(y)) : y \in Y, \mu_K(y) \in [0, 1]\} \tag{10}$$

and $w(d_i,c_k) = (y,\mu_K(y))$: $y \in Y$, $\mu_K(y) \in [0,1]$ – stands for a fuzzy linguistic weight given by the ith decision maker to kth criterion, with set of considerations Y being a set of possible fuzzy criterion weights.

To evaluate a variant using several various criteria requires a standardisation of individual evaluations, i.e. reducing them to a single common scale. With subjective evaluations expressed using linguistic variables, things are easier because it is sufficient for standardising evaluations that all such linguistic variables are defined over the same set of considerations X. For objective evaluations, the classic method of standardisation can be used, which consists in reducing evaluations to range [0,1] using the following relationships:

– for criteria aimed at maximisation:

$$\widehat{vo}(d_i,a_j,c_k)=\frac{vo(d_i,a_j,c_k)-\min_{j=1,\dots,a}(vo(d_i,a_j,c_k))}{\max_{j=1,\dots,a}(vo(d_i,a_j,c_k))-\min_{j=1,\dots,a}(vo(d_i,a_j,c_k))} \quad (11)$$

– and for criteria aimed at minimisation:

$$\widehat{vo}(d_i,a_j,c_k)=\frac{\max_{j=1,\dots,a}(vo(d_i,a_j,c_k))-vo(d_i,a_j,c_k)}{\max_{j=1,\dots,a}(vo(d_i,a_j,c_k))-\min_{j=1,\dots,a}(vo(d_i,a_j,c_k))} \quad (12)$$

To standardise between objective and subjective criteria using relationships (11) and (12), it is sufficient that for subjective criteria set of considerations X is equal to a non-fuzzy set of real numbers from range [0,1]. However, adopting such a set of considerations X may, in practice, make it difficult to define linguistic evaluations for subjective criteria. When accepting set of considerations X as range $[0,x_k]$, where $x_k > 1$, which is a typical way of proceeding in such situations, it is necessary to perform the second step of standardisation that involves reducing objective evaluations also to range $[0,x_k]$:

$$\overline{vo}(d_i,a_j,c_k) = x_k \cdot \widehat{vo}(d_i,a_j,c_k) \quad (13)$$

Hence, the ith decision maker formulates his or her evaluation of the jth variant as follows:

$$u(d_i,a_j) = \sum_{k=1}^{co} w(d_i,c_k)\cdot\overline{vo}(d_i,a_j,c_k) + \sum_{k=co+1}^{co+cs} w(d_i,c_k)\cdot vs(d_i,a_j,c_k) \quad (14)$$

Eq. (14) defines the weighted sum of objective and subjective criteria together. As a result we obtain a uniform, normalised assessment of a variant from the point of view of one decision maker. For specific forms of membership functions μ_K and μ_B, every time it is necessary to define the sense of an expression being the product of a real number and a fuzzy value (for objective criteria) and that of two fuzzy values (for subjective criteria).

When considering group decision-making, it should be taken into account that each decision maker can assign a different weight to the kth criterion. Then such weights should be aggregated, and aggregated weights of criteria should be determined for all decision makers. Let us define the aggregated weight function:

$$wa:\ (CO \cup CS) \to K \quad (15)$$

whereas

$$wa(c_k) = \frac{1}{d}\sum_{i=1}^{d} w(d_i,c_k) \quad (16)$$

Weights $w(d_i,c_k)$ are of fuzzy nature, so for each adopted membership function μ_K the form of relationship (16) has to be determined individually. Section 4, where an example of using the method is presented, shows how to determine this relationship for a trapezoidal membership function.

Once the aggregated weights of criteria are available, a summarised score of the jth variant can be calculated for a group of the decision makers under consideration.

$$ua(a_j) = \sum_{k=1}^{co}\left(wa(c_k)\cdot\sum_{i=1}^{d}\overline{vo}(d_i,a_j,c_k)\right) + \sum_{k=co+1}^{co+cs}\left(wa(c_k)\cdot\sum_{i=1}^{d} vs(d_i,a_j,c_k)\right) \quad (17)$$

As with formula (14), every time the operation of the product of a real number and a fuzzy value and that of two fuzzy values requires the adopted forms of characteristic functions μ_K and μ_B to be stated more precisely. Example in Section 4 defines these operations for trapezoidal membership functions (formulas (35)–(37)).

The group selection of the best variant, for which a fuzzy value of aggregated score ua is available, boils down to finding the maximum value of such a score. In general, searching for a maximum value leads to comparing fuzzy numbers, which may be difficult and ambiguous. A particular difficulty lies in the determination of how much fuzzy number M is greater than fuzzy number N. If both fuzzy numbers M and N belong to set of real numbers $\mathbb{R}$, the degree to which fuzzy number M is greater than fuzzy number N is denoted with $v(M > N)$ and defined as (Kacprzyk, 1986)

$$v(M > N) = \bigvee_{\substack{x>y\\ x,y\in\mathbb{R}}}(\mu_M(x)\wedge\mu_N(y)) \quad (18)$$

where

$$\bigvee_{x\in X} f(x) = \max_{x\in X} f(x) \quad (19)$$

and

$$a \wedge b = \min\quad (a,b) = \begin{cases} a & \text{for } a < b \\ b & \text{for } b \leqslant a \end{cases} \quad (20)$$

As fuzzy numbers cannot be compared unambiguously, non-fuzzy values representing fuzzy sets will finally be used for comparing variants. They are obtained using defuzzification. It is especially needed when we want to get strict arrangement of variants. In the cases where differences in the final assessments are small, using only fuzzy evaluations (linguistic values) we may get indistinguishable results. However, the use of defuzzification lets us to order the variants unambiguously according to group preferences, and simultaneously to express the strength of preferences. Which form of defuzzification function to choose is arbitrary. The calculation example presented in Section 4 uses defuzzification through bisection.

Hence, when the non-fuzzy value obtained via defuzzification, which represents fuzzy score $ua(a_j)$, is denoted with $\overline{\overline{ua}}(a_j)$, group preferred variant a^* fulfils relationship

$$a^* = a_r:\ (\overline{\overline{ua}}(a_r) = max(\overline{\overline{ua}}(a_j),\ j = 1,\dots,a)) \quad (21)$$

In addition to determining the best variant, the proposed method enables to arrange all variants, and the group preference relation fulfils relationship

$$O^* = \Big(a_r : (\overline{\overline{ua}}(a_r) = (max(\overline{\overline{ua}}(a_j),\ j = 1,\dots,a))) \succ \cdots \succ a_s: (\overline{\overline{ua}}(a_s) > \overline{\overline{ua}}(a_t)) \succ a_t \succ \cdots \succ a_w: (\overline{\overline{ua}}(a_w) = (min(\overline{\overline{ua}}(a_j),\ j = 1,\dots,a))\Big)\Big) \quad (22)$$

A positive side effect of using this method is learning the preference relations of individual decision makers. As with the group evaluation, the individual fuzzy score of the jth variant given by ith decision maker $u(d_i,a_j)$ using formula (14) has to be defuzzified here. The so obtained non-fuzzy evaluations are denoted with $\bar{\bar{u}}(d_i,a_j)$, and the individual preference relation of the ith decision maker is calculated using the following formula:

$$O_i = (a_r: (\bar{\bar{u}}(d_i,a_r) = (max(\bar{\bar{u}}(d_i,a_j),\ j = 1,\dots,a))) \succ \cdots \succ a_s: (\bar{\bar{u}}(d_i,a_s) > \bar{\bar{u}}(d_i,a_t)) \succ a_t \succ \cdots \succ a_w: (\bar{\bar{u}}(d_i,a_w) = (min(\bar{\bar{u}}(d_i,a_j),\ j = 1,\dots,a)))) \quad (23)$$

In this method, during the construction of the knowledge base, the decision maker is not obliged to express his/her individual preference relation explicitly. It is obtained by using formula (23). At this point a verification of the provided data is possible. The decision maker discovers the preference relation that results from his/her assessments of the variants and is able to check its correctness.

4. Use of the method to identify the causes of an aircraft incident

To illustrate the proposed method, we will consider an example of group decision making by a team investigating the causes of an aircraft incident. Such a team is composed of specialists in various fields, e.g. aircraft designers, medical practitioners, air traffic controllers, mechanical engineers and aviation law specialists, who are supposed to determine the facts of an incident. Such analysis is done to find out to what extent a certain category could lead to the occurrence of an aircraft incident. Then the experts hold a meeting to work out a group decision regarding the major causes of an incident. A recommendation how to prevent the reoccurrence of such causes is formulated.

The example relates to incident No. 291/05, which took place at the Chopin Airport in Warsaw under winter conditions in December 2005. During takeoff of a Boeing 757-200 (B757) aircraft the crew felt a shock and decided to abort the takeoff. The crew of another aircraft noticed fire in the B757's left engine. An ensuing inspection did not reveal any damage to the engine, and the pilot decided to repeat the takeoff. However, due to heavy traffic at the airport, the waiting time until the takeoff could be repeated was about 45 min. For that reason, the pilot decided to consult the aircraft engineer (which he failed to do previously). After the consultation he decided to cancel the flight. A closer examination of the engine showed damage that was very likely to make the repeated takeoff lead to an aircraft accident. A more detailed model of this incident using coloured Petri nets is presented in (Skorupski, 2013). Field experts have found out that the incident occurred due to the following errors:

- apron not entirely cleared of snow and ice,
- release of the parking brake when engaging the thrust automaton,
- blades of the left engine's compressor not thoroughly examined before flight,
- decision to continue the flight without consulting the aircraft engineer.

The decision problem considered in this paper relates to determining the significance of the causes leading to a serious aircraft incident.

Let us assume that the decision regarding the causes of the incident is based on the opinions of three experts, i.e.

$$D = \{d_1, d_2, d_3\}, \quad d = 3 \tag{24}$$

The submitted assessments point out to four possible causes, out of which one should be selected as the main cause of the incident. It is

Table 2
Parameters (m, n, p, q) of the trapezoidal membership function for the variables describing subjective criteria.

Degree of negligence, repeatability				
	m	*n*	*p*	*q*
Low	0	0	1	2
Medium	1	2	3	4
High	3	4	5	5

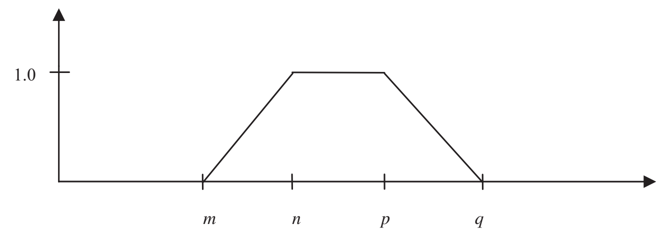

Fig. 1. Trapezoidal membership function with parameters (m, n, p, q).

also important to arrange the remaining causes by their significance in leading to the incident. So the set of variants can be defined as follows:

$$A = \{a_1, a_2, a_3, a_4\}, \quad a = 4 \tag{25}$$

where a_1 stands for the runway not entirely cleared of snow, a_2 – the technical support personnel's error of not detecting fatigue damage of the engine turbine blades, a_3 – the pilot's error of releasing the brake when engaging the thrust automaton, a_4 – the pilot's error of deciding to take off without consulting the aircraft engineer.

Let us adopt one objective criterion

$$CO = \{c_1\}, \quad co = 1 \tag{26}$$

where c_1 stands for the time that was available for making a key decision in a variant, e.g. to release a component for operation or select a flying technique. This value is a measure of how a mistake is likely to be prevented. A lower value of c_1 in a variant makes it more likely to be selected as the most important factor leading to an incident.

Let us adopt two subjective criteria

$$CS = \{c_2, c_3\}, \quad cs = 2 \tag{27}$$

where c_2 stands for the degree of negligence (error) made by the person responsible for the proper completion of a task and c_3 – repeatability of a cause in incidents of the same type.

Experts assign scores to individual variants. The effects of such assignments are presented in Table 1.

The evaluations of objective criterion c_1 are expressed in time units (here in seconds) and those of subjective criteria c_2 and c_3 are determined using a linguistic variable taking one of the three

Table 1
Evaluations of the variants made by the decision makers: $vo(d_i,a_j,c_k)$, $vs(d_i,a_j,c_k)$.

	Variant 1 (a_1)	Variant 2 (a_2)	Variant 3 (a_3)	Variant 4 (a_4)
Decision maker 1, criterion 1 (d_1,c_1)	20	15	10	12
Decision maker 2, criterion 1 (d_2,c_1)	23	19	8	15
Decision maker 3, criterion 1 (d_3,c_1)	19	21	11	14
Decision maker 1, criterion 2 (d_1,c_2)	*High*	*Low*	*High*	*Medium*
Decision maker 2, criterion 2 (d_2,c_2)	*High*	*Medium*	*High*	*High*
Decision maker 3, criterion 2 (d_3,c_2)	*Medium*	*High*	*Medium*	*Medium*
Decision maker 1, criterion 3 (d_1,c_3)	*Medium*	*High*	*Low*	*Medium*
Decision maker 2, criterion 3 (d_2,c_3)	*High*	*High*	*Medium*	*Low*
Decision maker 3, criterion 3 (d_3,c_3)	*Low*	*High*	*Low*	*Medium*

Table 3
Parameters (m, n, p, q) of the trapezoidal membership function for the variables describing the weights of the criteria.

Weight of the criteria	m	n	p	q
Not important	0	0	1	2
Not much important	1	2	3	4
Of average importance	3	4	5	6
Important	5	6	7	8
Very important	7	8	9	9

values: *low*, *medium* and *high*. Let us assume that the fuzzy sets relating to the individual values of linguistic variables are represented by trapezoidal membership functions defined over a set of considerations $X = [0,5]$ using formulas (28)–(30), the form of which is shown in Fig. 1, and parameters m, n, p, q are defined in Table 2.

$$\mu_{low}(x; m, n, p, q) = \begin{cases} 0, & x < m = n \\ 1, & n \leqslant x \leqslant p \\ \frac{q-x}{q-p}, & p < x \leqslant q \\ 0, & x > q \end{cases} \quad (28)$$

$$\mu_{medium}(x; m, n, p, q) = \begin{cases} 0, & x \leqslant m \\ \frac{x-m}{n-m}, & m < x \leqslant n \\ 1, & n < x \leqslant p \\ \frac{q-x}{q-p}, & p < x \leqslant q \\ 0, & x > q \end{cases} \quad (29)$$

$$\mu_{high}(x; m, n, p, q) = \begin{cases} 0, & x \leqslant m \\ \frac{x-m}{n-m}, & m < x \leqslant n \\ 1, & n < x \leqslant p \\ 0, & x > p = q \end{cases} \quad (30)$$

Let us assume that the decision makers can assign to the individual criteria fuzzy weights described using the linguistic variables: *not important*, *not much important*, *of average importance*, *important*, *very important*. These variables will be described using the trapezoidal membership functions of the fuzzy sets with the parameters shown in Table 3.

Table 6
Aggregated weights of the criteria assigned by the decision makers.

	Criterion 1 (c_1)	Criterion 2 (c_2)	Criterion 3 (c_3)
Weight $wa(c_k)$	(0,2,3,6)	(1,4,5,8)	(1,4,5,8)

The assignment of weights to individual criteria c_k by each decision maker d_i is presented in Table 4.

Another step is the standardisation of the evaluations of individual variants. First let us do it for the objective criterion using formulas (12) and (13). For the subjective criteria, no other transformation is needed because set of considerations $X = [0,5]$, similar to the value range of the objective criterion after standardisation. For all the criteria (after standardisation of the objective criterion), we strive to maximise them. The evaluations after standardisation are shown in Table 5.

Now let us calculate the aggregated weight of individual criteria $wa(c_k)$ using formula (16). As mentioned in Section 3, it is necessary here to define the sense of this operation in the context of particular membership function μ_K. For each criterion c_k, we obtain set of d weights assigned by the decision makers to the criteria. For the trapezoidal membership functions, we have to aggregate d sets described using parameters $(m(w(d_i,c_k)), n(w(d_i, c_k)), p(w(d_i,c_k)), q(w(d_i,c_k)))$ then. Notation $m(w(d_i,c_k))$ should be interpreted as the first parameter of the trapezoidal membership function for the linguistic variable assigned by the d_ith decision maker as the weight of the c_kth evaluation criterion. Aggregated weight $wa(c_k)$ takes the form of a trapezoidal membership function with parameters (m^1, n^1, p^1, q^1) defined as follows:

$$m^1 = \min_{i=1..d}(m(w(d_i, c_k))) \quad (31)$$

$$n^1 = \frac{1}{d}\sum_{i=1}^{d} n(w(d_i, c_k)) \quad (32)$$

$$p^1 = \frac{1}{d}\sum_{i=1}^{d} p(w(d_i, c_k)) \quad (33)$$

$$q^1 = \max_{i=1..d}(q(w(d_i, c_k))) \quad (34)$$

In the analysed example, the aggregated weights of the criteria, determined using formulas (31)–(34), are shown in Table 6.

Table 4
Weights of the criteria assigned by the decision makers: $w(d_i,c_k)$.

	Criterion 1 (c_1)	Criterion 2 (c_2)	Criterion 3 (c_3)
Decision maker 1 (d_1)	*Not much important*	*Of average importance*	*Of average importance*
Decision maker 2 (d_2)	*Not important*	*Not much important*	*Important*
Decision maker 3 (d_3)	*Of average importance*	*Important*	*Not much important*

Table 5
Standardised evaluations of the variants.

	Variant 1 (a_1)	Variant 2 (a_2)	Variant 3 (a_3)	Variant 4 (a_4)
Decision maker 1, criterion 1 (d_1,c_1)	0	2.5	5	4
Decision maker 2, criterion 1 (d_2,c_1)	0	1.33	5	2.67
Decision maker 3, criterion 1 (d_3,c_1)	1	1.33	5	3.5
Decision maker 1, criterion 2 (d_1,c_2)	(3,4,5,5)	(0,0,1,2)	(3,4,5,5)	(1,2,3,4)
Decision maker 2, criterion 2 (d_2,c_2)	(3,4,5,5)	(1,2,3,4)	(3,4,5,5)	(3,4,5,5)
Decision maker 3, criterion 2 (d_3,c_2)	(1,2,3,4)	(3,4,5,5)	(1,2,3,4)	(1,2,3,4)
Decision maker 1, criterion 3 (d_1,c_3)	(1,2,3,4)	(3,4,5,5)	(0,0,1,2)	(1,2,3,4)
Decision maker 2, criterion 3 (d_2,c_3)	(3,4,5,5)	(3,4,5,5)	(1,2,3,4)	(0,0,1,2)
Decision maker 3, criterion 3 (d_3,c_3)	(0,0,1,2)	(3,4,5,5)	(0,0,1,2)	(1,2,3,4)

Table 7
Summarised evaluation of the variants.

	Variant 1 (a_1)	Variant 2 (a_2)	Variant 3 (a_3)	Variant 4 (a_4)
(c_1)	(0,2,3,6) · 1 = (0,2,3,6)	(0,2,3,6) · 3.83 = (0,7.67,11.5,23)	(0,2,3,6) · 15 = (0,30,45,90)	(0,2,3,6) · 10.17 = (0,20.33,30.5,61)
(c_2)	(1,4,5,8) · (7,10,13,14) = (7,40,65,112)	(1,4,5,8) · (4,6,9,11) = (4,24,45,88)	(1,4,5,8) · (7,10,13,14) = (7,40,65,112)	(1,4,5,8) · (5,8,11,13) = (5,32,55,104)
(c_3)	(1,4,5,8) · (4,6,9,11) = (4,24,45,88)	(1,4,5,8) · (9,12,15,15) = (9,48,75,120)	(1,4,5,8) · (1,2,5,8) = (1,8,25,64)	(1,4,5,8) · (2,4,7,10) = (2,16,35,80)
Total $ua(a_j)$	(11,66,113,206)	(13,79.67,131.5,231)	(8,78,135,266)	(7,68.33,120.5,245)

Table 8
Results of the defuzzification through bisection.

	Variant 1 (a_1)	Variant 2 (a_2)	Variant 3 (a_3)	Variant 4 (a_4)
$\overline{\overline{ua}}(a_j)$	*99*	113.79	121.75	110.21

Now the individual variants have to be assessed taking into account the aggregated weight of each criterion. Before doing this, it is still necessary (as mentioned in Section 3) to interpret the operation of multiplying a fuzzy number by a constant and that of adding up and multiplying two fuzzy numbers by themselves. For the trapezoidal membership functions with parameters (m_1, n_1, p_1, q_1) and (m_2, n_2, p_2, q_2) and constant s, these operations are to be defined as follows (Tyagi, Pandey, & Tyagi, 2010):

$$s \cdot (m_1, n_1, p_1, q_1) = (s \cdot m_1, s \cdot n_1, s \cdot p_1, s \cdot q_1) \quad (35)$$

$$(m_1, n_1, p_1, q_1) \cdot (m_2, n_2, p_2, q_2) = (m_1 \cdot m_2, n_1 \cdot n_2, p_1 \cdot p_2, q_1 \cdot q_2) \quad (36)$$

$$(m_1, n_1, p_1, q_1) + (m_2, n_2, p_2, q_2) = (m_1 + m_2, n_1 + n_2, p_1 + p_2, q_1 + q_2) \quad (37)$$

Hence, the summarised evaluation of the individual variants made by all the decision makers is presented in Table 7.

The last step of the method is to select the variant with the highest evaluation score. A defuzzification method such as bisection can be used for this purpose. The value that divides the area under curve μ_B into halves is selected as that which represents fuzzy set $\bar{u}$, i.e.

$$\int_{u_{min}}^{\bar{u}} \mu_B(u)du = \int_{\bar{u}}^{u_{max}} \mu_B(u)du \quad (38)$$

For the trapezoidal membership function with parameters (m, n, p, q), value $\bar{u}$ can be calculated using the formula

$$\bar{u} = \frac{1}{4}(m + n + p + q) \quad (39)$$

The results obtained are shown in Table 8.

Finally, the highest score was assigned to variant a_3 and a slightly lower one to variant a_2. The next rank is variant a_4, and variant a_1 is least preferred. It can be said then that the preference relation obtained for all the decision makers, which is equivalent to the group decision made as part of a multi-criteria evaluation under uncertainty conditions, is defined as follows:

$$O^* = (a_3 \succ a_2 \succ a_4 \succ a_1) \quad (40)$$

It can still be analysed which variants were preferred by the individual decision makers. Let us name it decision maker's induced preference. For this purpose, we will use formula (14). The results of the fuzzy evaluations in the form of a trapezoidal membership function are presented in Table 9.

By using formula (39) for defuzzification through bisection as in the previous case, we obtain the decision makers' non-fuzzy evaluations of the variants. The results are presented in Table 10.

Based on Table 10 and formula (23) it is possible to determine the form of preference relations between the variants for each decision maker.

$$O_1 = (a_3 \succ a_4 \succ a_1 \succ a_2) \quad (41)$$

$$O_2 = (a_1 \succ a_2 \succ a_3 \succ a_4) \quad (42)$$

$$O_3 = (a_3 \succ a_4 \succ a_2 \succ a_1) \quad (43)$$

As it can be seen, none of the decision makers had the same preference relation as the group decision obtained using the presented method. In addition, it is worth mentioning that the group preference arrangement obtained using the aforesaid method described in (Kacprzyk, 1986) is different from that obtained using the method presented in this paper.

Determining and presenting the induced preference relations of individual decision makers may persuade a decision maker to change his or her original evaluations. It also enables to assess the sensitivity of an obtained solution to the decision maker's change of his or her mind.

Let us assume that in the presented example decision maker d_1, after his or her induced preference relation O_1 has been presented to him or her, comes to the conclusion that after all he or she prefers variant a_2 to a_1. So he or she decides to take a closer look at the evaluations he or she assigned previously and change evaluation

Table 9
Fuzzy evaluations of the variants made by the individual decision makers $u(d_i,a_j)$.

	Variant 1 (a_1)	Variant 2 (a_2)	Variant 3 (a_3)	Variant 4 (a_4)
Decision maker 1 (d_1)	(12,24,40,54)	(11.5,21,37.5,52)	(14,26,45,62)	(10,24,42,64)
Decision maker 2 (d_2)	(18,32,50,60)	(16,28,45.33,58.67)	(8,20,41,62)	(3,8,24.67,41.33)
Decision maker 3 (d_2)	(5,12,24,40)	(18,32,50,60)	(20,32,49,70)	(16.5,30,47.5,69)

Table 10
Non-fuzzy evaluations of the variants obtained using defuzzification $\bar{u}(d_i, a_j)$.

	Variant 1 (a_1)	Variant 2 (a_2)	Variant 3 (a_3)	Variant 4 (a_4)
Decision maker 1 (d_1)	32.5	30.5	36.75	35
Decision maker 2 (d_2)	40	37	32.75	19.25
Decision maker 3 (d_2)	20.25	40	42.75	40.75

Table 11
Results of defuzzification after changing evaluation $vs(d_1,a_1,c_2)$ to *medium*.

	Variant 1 (a_1)	Variant 2 (a_2)	Variant 3 (a_3)	Variant 4 (a_4)
$\overline{\overline{ua}}(a_j)$	92	113.79	121.75	110.21

$vs(d_1,a_1,c_2)$ from *high* to *medium* in order to weaken the significance of variant a_1. The results of the final evaluation of the variants after such modification are shown in Table 11.

As compared with Table 8, the general evaluation of variant a_1 was changed, which is as expected. At the same time, the group preference relations between variants O^* were not changed. However, the induced preference relation of decision maker d_1 was changed according to his or her judgment and intention. After modifying evaluation $vs(d_1,a_1,c_2)$ to *medium*, the non-fuzzy evaluation of variant a_1 by decision maker d_1, $\bar{u}(d_1, a_1) = 25$, whereas the induced preference relation of decision maker d_1 is equal to

$$O'_1 = (a_3 \succ a_4 \succ a_2 \succ a_1) \tag{44}$$

5. Summary

The problem of group decision making has already been reported in the literature for many years. The proposed method addresses such decision when there are many evaluation criteria, classified into one of the groups: objective (expressed using non-fuzzy values) and subjective (expressed using linguistic variables) criteria. Its idea is based on the assumption that group evaluation takes place when decision makers cannot determine precisely individual preferences between variants of choice.

The proposed method shows several advantages over the approach used so far. Firstly, it enables to find a solution without negotiations between decision makers. This approach is of advantage when there is a risk that some experts will be dominated by others or discussion between experts will not be possible, e.g. due to time or distance constraints.

Secondly, the method enables experts to express the strength of preferences between alternatives. They do not have to make it explicitly, however, but only by stating the importance (weights) of individual criteria. Both the importance of criteria and evaluations assigned to individual variants in terms of such criteria can be defined using fuzzy expressions. The method takes into account the strength of preferences in analysis while reflecting the decision maker's conviction about the significance of certain criteria to the final solution.

Thirdly, the method makes it possible not only to find a single preferred solution but also create group preference relation O^*, which in itself is a desirable feature of the solution to a problem. It also enables to determine how much individual variants are superior to others, which makes it possible to notice e.g. variants that are nearly indifferent.

Fourthly, it is possible to reproduce the actual preference relations of individual decision makers based on how important criteria are to them and what scores they give to the evaluations assigned to alternatives.

Fifthly, making decision makers aware of their actual preferences which originally they could not define precisely may be a desirable element of analysis. This is so because they may be induced to change how they evaluate the weights of criteria or how they score the evaluations assigned to variants. The method makes it possible to find a new solution when a decision maker modifies the weight of any of the criteria for evaluation of variants.

Sixthly, the method takes into account the uncertainty of decision making. For many problems, including those of high complexity, each decision maker does not have full knowledge. The presented example of analysing the causes of a serious aircraft incident is a classic example where a group of experts is required to have a very specific and detailed yet interdisciplinary knowledge. Obviously, each expert has only a fraction of such knowledge. Another aspect of taking into consideration uncertainty lies in that experts can make fuzzy evaluations when they are not sure about them or it is not possible to gain full knowledge, e.g. due to a lack of complete measurement data. In this particular example under consideration, the analysis showed that the factor that most contributed to the incident was a pilot's error, who released the parking brake when engaging the thrust automaton. This factor was mentioned as a circumstance favouring the incident in the report of the Investigation Commission. It was also the basis for a prophylactic recommendation, which is consistent with the findings of the paper.

Further research will focus on two aspects. The first will consist in building a computer system that enables a group decision making, despite lack of complete competence of each of domain experts. It will use the model of inference engine presented in this paper. In addition, it is planned to develop an improved method for building a knowledge base for the expert system. It is about automating the process of changing the evaluation of the weights of criteria and scores of the evaluations assigned to variants, carried out on the basis of individual induced preference relations. The second will consist in determining the compliance of obtained group solution with individual decision makers' assessments. For this purpose, fuzzy regret matrix will be used.

References

Chaudhuri, S., & Deb, K. (2010). An interactive evolutionary multi-objective optimization and decision making procedure. *Applied Soft Computing, 10*, 496–511.

Chin, K., & Fu, C. (2014). Integrated evidential reasoning approach in the presence of cardinal and ordinal preferences and its applications in software selection. *Expert Systems with Applications, 41*, 6718–6727.

Cho, K. T. (2003). Multicriteria decision methods: An attempt to evaluate and unify. *Mathematical and Computer Modelling, 37*, 1099–1119.

Chuu, S. (2011). Interactive group decision-making using a fuzzy linguistic approach for evaluating the flexibility in a supply chain. *European Journal of Operational Research, 213*, 279–289.

Cook, W. D., Golany, B., Penn, M., & Raviv, T. (2007). Creating a consensus ranking of proposals from reviewers' partial ordinal rankings. *Computers & Operations Research, 34*, 954–965.

de Wit, F. R. C., Jehn, K. A., & Scheepers, D. (2013). Task conflict, information processing, and decision-making: The damaging effect of relationship conflict. *Organizational Behavior and Human Decision Processes, 122*, 177–189.

Dheena, P., & Mohanraj, G. (2011). Multicriteria decision-making combining fuzzy set theory, ideal and anti-ideal points for location site selection. *Expert Systems with Applications, 38*, 13260–13265.

Dubois, D., & Prade, H. (1992). On the relevance of non-standard theories of uncertainty in modeling and pooling expert opinions. *Reliability Engineering & System Safety, 36*, 95–107.

Greco, S., Matarazzo, B., & Slowinski, R. (2001). Rough sets theory for multicriteria decision analysis. *European Journal of Operational Research, 129*, 1–47.

Guha, D., & Chakraborty, D. (2011). Fuzzy multi attribute group decision making method to achieve consensus under the consideration of degrees of confidence of experts' opinions. *Computers & Industrial Engineering, 60*, 493–504.

Hamouda, L., Kilgour, D. M., & Hipel, K. W. (2006). Strength of preference in graph models for multiple-decision-maker conflicts. *Applied Mathematics and Computation, 179*, 314–327.

Kacprzyk, J. (1986). *Fuzzy Sets in Systems Analysis*. Warszawa: Państwowe Wydawnictwo Naukowe.

Kocher, M., Strauß, S., & Sutter, M. (2006). Individual or team decision-making—Causes and consequences of self-selection. *Games and Economic Behavior, 56*, 259–270.

Merigó, J. M., Casanovas, M., & Yang, J. (2014). Group decision making with expertons and uncertain generalized probabilistic weighted aggregation operators. *European Journal of Operational Research, 235*, 215–224.

Perez, I. J., Cabrerizo, F. J., Alonso, S., & Herrera-Viedma, E. (2014). A new consensus model for group decision making problems with non-homogeneous experts. *IEEE Transactions on Systems, Man, and Cybernetics: Systems, 44*(4), 494–498.

Rao, R. V., & Patel, B. K. (2010). A subjective and objective integrated multiple attribute decision making method for material selection. *Materials & Design, 31*, 4738–4747.

Rose, J., & Hensher, D. A. (2004). Modelling agent interdependency in group decision making. *Transportation Research Part E: Logistics and Transportation Review, 40*, 63–79.

Rosmuller, N., & Beroggi, G. E. G. (2004). Group decision making in infrastructure safety planning. *Safety Science, 42*, 325–349.
Skorupski, J. (2013). Traffic incidents analysis as a tool for improvement of transport safety. In A. Weintrit (Ed.), *Navigational problems. Marine navigation and safety of sea transportation* (pp. 101–108). Leiden: CRC Press/Taylor & Francis/Balkema.
Stewart, T. J., French, S., & Rios, J. (2013). Integrating multicriteria decision analysis and scenario planning—Review and extension. *Omega, 41*, 679–688.
Tavana, M., Khalili-Damghani, K., & Abtahi, A. (2013). A hybrid fuzzy group decision support framework for advanced-technology prioritization at NASA. *Expert Systems with Applications, 40*, 480–491.
Tsai, R., & Böckenholt, U. (2006). Modelling intransitive preferences: A random-effects approach. *Journal of Mathematical Psychology, 50*, 1–14.
Tyagi, S. K., Pandey, D., & Tyagi, R. (2010). Fuzzy set theoretic approach to fault tree analysis. *International Journal of Engineering, Science and Technology, 2*, 276–283.
Wibowo, S., & Deng, H. (2013). Consensus-based decision support for multicriteria group decision making. *Computers & Industrial Engineering, 66*, 625–633.
Wu, J., & Chiclana, F. (2012). Non-dominance and attitudinal prioritisation methods for intuitionistic and interval-valued intuitionistic fuzzy preference relations. *Expert Systems with Applications, 39*, 13409–13416.
Xiong, J., Tan, X., Yang, K., & Chen, Y. (2013). Fuzzy group decision making for multiobjective problems: tradeoff between consensus and robustness. *Journal of Applied Mathematics, 2013*, 9.
Yousefi, A., & Hadi-Vencheh, A. (2010). An integrated group decision making model and its evaluation by DEA for automobile industry. *Expert Systems with Applications, 37*, 8543–8556.
Yu, D., Zhang, W., & Xu, Y. (2013). Group decision making under hesitant fuzzy environment with application to personnel evaluation. *Knowledge-Based Systems, 52*, 1–10.
Yue, Z. (2012). Extension of TOPSIS to determine weight of decision maker for group decision making problems with uncertain information. *Expert Systems with Applications, 39*, 6343–6350.
Zgurovsky, M. Z., Totsenko, V. G., & Tsyganok, V. V. (2004). Group incomplete paired comparisons with account of expert competence. *Mathematical and Computer Modelling, 39*, 349–361.
Zhang, B., & Dong, Y. (2013). Consensus rules with minimum adjustments for multiple attribute group decision making. *Procedia Computer Science, 17*, 473–481.
Zhang, D., Zhang, J., Lai, K., & Lu, Y. (2009). An novel approach to supplier selection based on vague sets group decision. *Expert Systems with Applications, 36*, 9557–9563.
Zhang, G., Dong, Y., & Xu, Y. (2012). Linear optimization modeling of consistency issues in group decision making based on fuzzy preference relations. *Expert Systems with Applications, 39*, 2415–2420.