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The simulation-fuzzy method of assessing the risk of air traffic accidents using the fuzzy risk matrix

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ABSTRACT

Modernization efforts in air transport are preceded by analyses to make sure that they do not lead to excessive risk in air operations. The risk assessment methods that have been applied so far are insufficient since their classification of combinations of both the probability and consequences of an event is too rough. The aim of this study was to present a risk assessment method that allows to express risk by using a continuous numeric scale. Therefore, a fuzzy risk matrix is proposed in which both the probability and severity of the consequences are expressed by linguistic variables while the risk assessment is made by the fuzzy inference system. A model based on Petri nets was used to assess the probability of aircraft collision, while computer-implemented expert knowledge in the form of fuzzy inference rules was used to estimate the consequences. Experiments carried out using this tool allowed to assess the risk of a Runway Incursion-type traffic incident transforming into an accident at a *tolerable* level. Furthermore, it was found that the transition of this assessment to the level of *intolerable* is possible when unfavorable visibility conditions occur in connection with the delayed reaction of several participants of the incident. The proposed method is general and can be applied in different areas.

1. Introduction

Air transport has constituted the safest mode of traveling for years. This is the result of extensive usage of risk analysis methods which affects the vast majority of actions performed in the field of air traffic management. Every change of the operating procedure, implementation of new equipment or software, or modification of training programs is preceded by risk analysis in order to check whether implementation of the planned change will maintain at least the current safety level. Safety management systems have been implemented in many areas in which continuous monitoring of the safety system is being conducted and corrective procedures are initiated if there are any irregularities (ICAO, 2012).

An important part of the safety management process is the assessment of risk. In most cases a scale that is discrete and consists of three levels is used. The risk can either be acceptable, tolerable, or intolerable. This assessment is based on fusing the probability of an adverse event with the severity of its consequences. The probability, as well as the consequences, can be estimated by using analytic methods. In practice, expert opinions are used as well. Regardless of the way in which the estimates are obtained, they are usually classified into one of five exclusive categories. The result of the classification is the risk matrix in which each element (i.e. each combination of the probability and severity of the consequences) is tied to one of the three risk assessments (ICAO, 2012).

However, the above practice seems to be inadequate. Assessments of probabilities and consequences are often imprecise but are considered to belong to one particular category that determines further risk assessment. Also, the assessments often come from experts and have a descriptive (qualitative) nature. Such knowledge is subjective and, what is more, the linguistic terms that are used can be understood in different ways by different experts. Therefore, it is not possible to unequivocally and precisely tie those assessments to one of the several possible categories; there is a whole range of intermediate situations and the classic risk matrix does not allow to take those situations into account.

In this paper we proposed to apply fuzzy sets to determine both the probability and the severity of the consequences of an event. This results in the creation of a fuzzy risk matrix which is in fact a fuzzy inference system having two inputs and one output. Thanks to applying this mechanism we obtain a risk assessment that is continuous in a predetermined numerical scale. It allows us to compare the risk for different situations, even when they belong to one particular category, e.g. *tolerable*. Such comparisons allow to rationalize actions as a part of risk management systems.

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1.1. Literature review

Risk assessment methods in air traffic were proposed in many papers, for instance, by Stroeve et al. (2009), Di Gravio et al. (2015), Tamasi and Demichela (2011), Lee (2006), Ale and Piers (2000), and Janic (2000). Risk assessment consists of estimating the probability of events (Taleb, 2007; Yang et al., 2015; Shyur, 2008; Wong et al., 2009) and their consequences (Wilke et al., 2014; Ayres et al., 2013). In the latter aspect, a particularly important question is how to take uncertainty into account (Aven and Zio, 2011; Aven, 2013, 2015; Johansen and Rausand, 2015; Skorupski, 2014, 2015b). A review of methods for risk analysis under deep uncertainty can be found in (Cox, 2012). A more general review of risk assessment methods in civil aviation can be found in (Netjasov and Janic, 2008).

The case study analyzed in this paper belongs to the category of Runway Incursion. It was also analyzed by Chang and Wong (2012), Wilke et al. (2015), Stroeve et al. (2013, 2015), Yang and Ziqi (2014), and Schönefeld and Möller (2012).

The method proposed in this paper aims to change the approach to safety management, i.e. from a reactive to a proactive approach. This was suggested earlier by Herrera et al. (2009), Sawyer et al. (2015), Ternov and Akselsson (2004), and Kontogiannis and Malakis (2009). The method of obtaining a proactive approach is to put more emphasis on an analysis of incidents instead of accidents. Specific methods to analyze incidents were suggested by Ali et al. (2015), Lower et al. (2016), Brooker (2005), and Nazeri et al. (2008).

Fuzzy inference systems have gained increasing popularity in risk analysis in many areas of technology (Kahraman et al., 2008; Yang and Wang, 2015; Saracino et al., 2015). Fuzzy expert systems for aviation risk assessment have been investigated in (Ken, 2013; Skorupski and Uchroński, 2015a, 2015b; Hadjimichael, 2009; Żurek and Grzesik, 2015).

The concept of the fuzzy risk matrix that was used in this research was proposed by Markowski and Mannan (2008) for the analysis of a distillation column unit. It was also used by Khaleghi et al. (2013), Liu et al. (2014), and Ataallahi and Shadizadeh (2015). In this paper, a proposal for its adaptation and expansion to the problem of risk analysis in air traffic is presented.

1.2. The concept of the paper

The practice of risk analysis shows that using the intuitive opinions of experts which are, by their nature, inaccurate and inexact, is inevitable. This refers to many applications but is especially visible in air traffic management, where socio-technical systems with the vital role of the human factor are widely used. At the same time, risk models and applied calculating procedures are based on classical mathematics, assuming that input data are precise and accurate, but this is often not true. In this paper we proposed to use mathematical models that are adequate for the high level of uncertainty that occurs in practice. There are many possible solutions. In this paper, an approach based on a fuzzy risk matrix is used. The main element is the fuzzy inference system with a hybrid knowledge base, partially obtained from experts and partially from measurement, and with models built on classical mathematics.

The elements of a risk matrix are: the probability of adverse event occurrence and the severity of consequences caused by this particular event. Estimation of the probability is done by simulation and will be exemplified by a model of an air traffic accident created with the use of Petri nets (Skorupski, 2015a). In turn, estimation of the consequences cannot be effectively algorithmized, especially when taking into account not only the total loss of equipment or the loss of life of all the exposed people (which often happens in air traffic accidents), but also smaller damages or injuries. Expert methods are necessary in such situations. Fusing the use of expert opinions and the discrete risk matrix seems inappropriate. The boundaries between particular assessments of input and output values should be fuzzified. Therefore, a fuzzy risk matrix is proposed.

The structure of the remaining part of the paper is as follows: Section 2 describes the general concept of the simulation-fuzzy risk assessment method; Section 3 presents a serious air traffic incident that will be analyzed using the proposed method. The primary elements of the model for assessment of the probability that an air traffic incident will transform into an accident, as elaborated in (Skorupski, 2015a), are also presented; Section 4 includes a description of all the local fuzzy inference systems used in the risk assessment; Section 5 contains a description of the created computer tool as well as a presentation of the simulation experiments performed using this tool; Section 6 includes a summary and the final conclusions.

2. The simulation-fuzzy method for assessing the risk of accident by using fuzzy inference

The proposed simulation-fuzzy risk assessment method is based on two pillars. The first is a simulation analysis of the probability of an accident. The second is a fuzzy analysis of its effects. Both pillars depend on the number of input parameters, both static and dynamic. The final step of the method is risk assessment using the fuzzy risk matrix implemented as a fuzzy inference system.

2.1. General structure of the model

As was stated above, two estimates are necessary to perform an assessment of the risk of an accident. Both of the estimates (the probability of the accident and the severity of its consequences), being the input to the fuzzy inference system implementing the fuzzy risk matrix, are linguistic variables represented by fuzzy sets. A linguistic variable is a variable whose values are either words or sentences in a natural or artificial language. These words or sentences will be called the linguistic values of a linguistic variable. Details are provided in particular sections which describe subsequent linguistic variables. Also, a graphical interpretation of particular values of each of the linguistic variables is presented. A fuzzy set will denote a set of

$$A = \{(x, \mu_A(x)) : x \in X, \mu_A(x) \in [0, 1]\} \quad (1)$$

where μ_A is the membership function of this set.

Schematically, the fuzzy inference system is presented in Fig. 1.

For the input of the fuzzification block we give unfuzzy values $\boldsymbol{X}$ obtained through observation or measurements. In the fuzzification block, based on the specified membership functions, they are associated with the linguistic variables. The fuzzy values $\widetilde{\boldsymbol{X}}$ constitute the input for the inference block. This block uses the base of fuzzy rules which in our case are created by experts, practitioners in the field of airport safety. The inference block, on the basis fuzzy prerequisites and all the fulfilled rules, specifies the

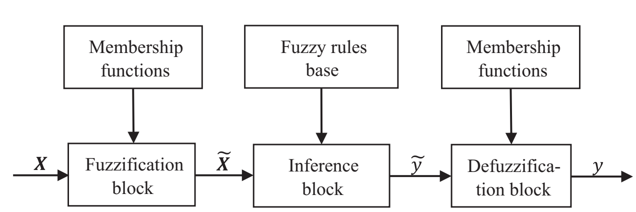

Fig. 1. General structure of the fuzzy inference system.

conclusion in the form of a linguistic variable $\tilde{y}$. This conclusion is an input for the defuzzification block which on the basis of the specified membership function associates the fuzzy value with the output unfuzzy value y. It constitutes the result of the operation of the fuzzy inference system.

The probability of the event may be obtained using one of the analytic or simulation methods. Using analytic methods may be more efficient in static cases, while in dynamic cases the use of simulation is proposed.

Estimating the consequences of an event by using formal methods is very difficult; this difficulty arises from the need to anticipate both the actions performed by the participants of an event in hazardous conditions and the impact of those actions on the scale of damage or human casualties. Taking all of this into account, the use of expert knowledge is proposed. The fuzzy inference system created on the basis of this knowledge will typically have a hierarchical structure.

The general structure of the proposed risk assessment model is presented in Fig. 2.

The input variable vectors $\boldsymbol{X}_i$ correspond to measureable parameters typical for a particular event, such as: distance, velocity of objects, and time necessary to perform an action, etc. By using fuzzy inference systems and based on expert knowledge one can transform them into anticipated actions $y_1,\ldots,y_n$ of the participants of the event. The same input variables $\boldsymbol{X}_i$ allow to calculate probability y_p of the occurrence of the analyzed adverse event. The participants' actions allow, in turn, to assess the severity of the consequences y_c. Again, obtaining expert knowledge is needed to achieve this. Estimates of probability y_p and consequences y_c allow to determine risk tolerance level z_r by using a fuzzy risk matrix implemented as the fuzzy inference system.

Fuzzy inference systems that lead to the determination of actions performed by particular participants and of the consequences of an event as well as the module used to assess the probability of an event must be determined separately for each particular event, including its details and nature; whereas a fuzzy inference system implementing a fuzzy risk matrix is more general and can be adopted for the whole class of events. Section 2.2 presents an example of this kind of analysis as applied to incidents and accidents in air traffic.

2.2. Fuzzy risk matrix

The general concept of risk analysis as proposed in this paper is consistent with the International Civil Aviation Organization Safety Management Manual (ICAO, 2012). It involves assigning the probability of an event along with its consequences to one of five available categories. The combination of the probability and consequences marked by a symbol (e.g. 3A, 4E, etc.) allows for risk assessment and to assign it to one of three categories: acceptable, tolerable, intolerable (Table 1).

The Risk Matrix, which is consistent with (ICAO, 2012), is widely used in risk analysis in air traffic. It has, however, quite significant drawbacks, the most important of which is the adoption of discrete values of the probability and the consequences. The adopted ranges of values allowing assignment to a particular category are often very wide, and sometimes they are not defined formally at all. This causes a situation where values located at the edge of each class may be classified as belonging to a contiguous class. This issue is problem-specific and externally context-dependent. In the practice of the Safety Management System (SMS) in air traffic, probability assessment is made by experts who usually do not want to conduct very strict evaluations and therefore determine them as located at the intersection of two categories. Unfortunately, the Risk Matrix (Table 1) does not allow to use such assessments. Therefore, it has been proposed to treat both the probability and the consequences evaluations as linguistic variables whose values are represented by fuzzy sets with adequate membership functions.

The proposed concept of the fuzzy risk matrix corresponds to the following fuzzy inference system. The input variables are:

1. The linguistic variable *Severity of consequences* describing the outcomes of an event. It will take the following values: *negligible*, *minor*, *major*, *hazardous*, *catastrophic*. A huge discrepancy is typical of expert assessments in the case of simultaneous occurrence of property damage and human casualties. Taking this fact into account, the proposed scale (Fig. 3) is characterized by a high level of fuzziness – there are large areas of overlap between the membership functions representing the linguistic variables describing the consequences of an air event. This allows to represent different "scales" of consequences, even those described with the same word (the same linguistic value), e.g. for the value 1.25 in the adopted numerical scale of consequences (Fig. 3) we are dealing with full membership ($\mu = 1$) in the value *minor* of the linguistic variable describing the severity of the consequences. But the same value 1.25 belongs, with a certain degree ($\mu = 0.25$), to the value *negligible* and also with the same degree ($\mu = 0.25$) to the value *major* of this linguistic variable. All of these intermediate values can represent the full spectrum of the possible consequences of aviation incidents, such as "minor wing collision" and "catastrophic collision".

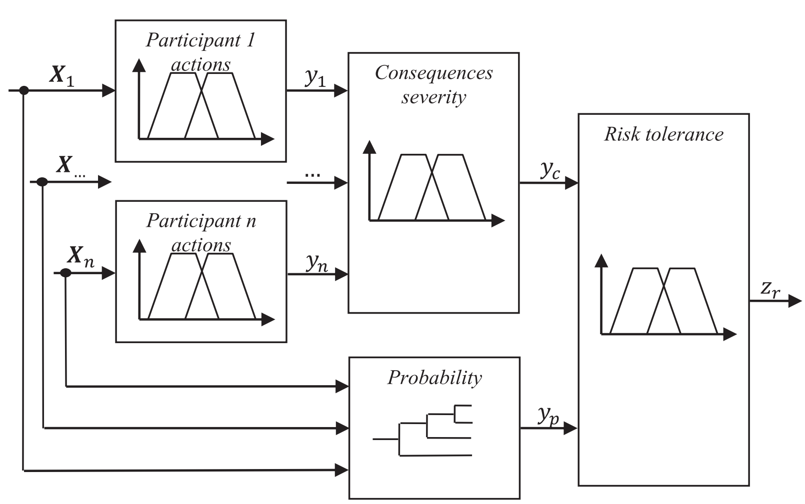

Fig. 2. Structure of the model for simulation-fuzzy risk assessment.

Table 1
Risk matrix according to ICAO Doc 9859 (ICAO, 2012).

		Consequences				
		Catastrophic (A)	Hazardous (B)	Major (C)	Minor (D)	Negligible (E)
Probability	Frequent (5)	intolerable	intolerable	intolerable	tolerable	tolerable
	Occasional (4)	intolerable	intolerable	tolerable	tolerable	tolerable
	Remote (3)	intolerable	tolerable	tolerable	tolerable	acceptable
	Improbable (2)	tolerable	tolerable	tolerable	acceptable	acceptable
	Extremely improbable (1)	tolerable	acceptable	acceptable	acceptable	acceptable

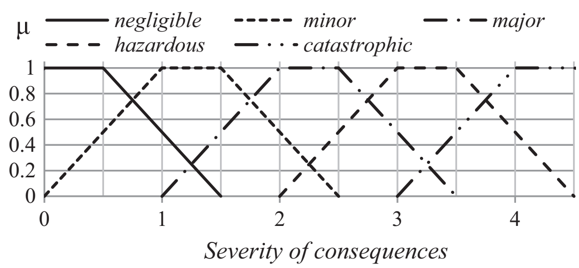

Fig. 3. Membership functions of the linguistic input variable *Severity of consequences*.

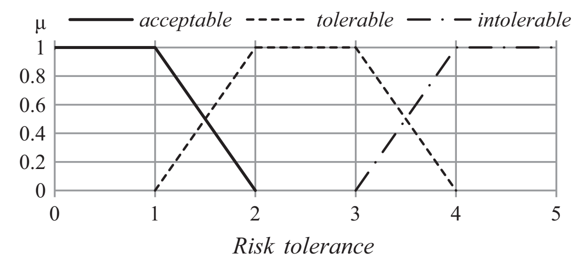

Fig. 5. Membership functions of the linguistic output variable *Risk tolerance*.

Table 2
Fuzzy inference rules for the linguistic variable *Risk tolerance*.

Rule number	*Probability*	*Severity of consequences*	*Risk tolerance*
6	*high*	*catastrophic*	*intolerable*
10	*high*	*negligible*	*tolerable*
11	*medium*	*catastrophic*	*intolerable*
14	*medium*	*minor*	*tolerable*
15	*medium*	*negligible*	*acceptable*
19	*small*	*minor*	*acceptable*

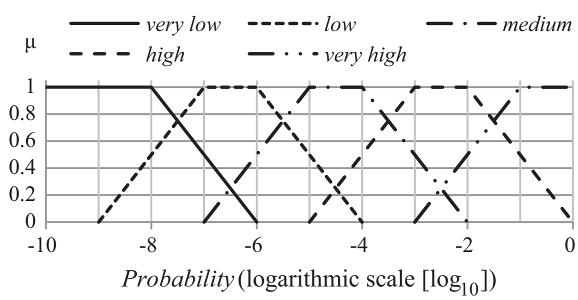

Fig. 4. Membership functions of the linguistic input variable *Probability*.

2. The linguistic variable *Probability* describing the probability of an event occurrence. It will also take five fuzzy values: *very small*, *small*, *medium*, *high*, *very high*. The scale proposed for this linguistic variable is based on a logarithmic scale introduced in (Lower et al., 2013). However, it was modified by extending its fuzziness range to map a wider degree of uncertainty that can be observed while assessing such kinds of events (Fig. 4).

In turn, the linguistic output variable *Risk tolerance* will describe an evaluation of risk in the same way as in (ICAO, 2012) and will take one of the three possible values: *acceptable*, *tolerable*, *intolerable*. In this paper, trapezoidal membership functions of fuzzy sets corresponding to the input and output linguistic values were taken. These are presented in Figs. 3–5.

The model is complemented by fuzzy inference rules representing risk assessment criteria. The general principle of creating these rules was adopted in accordance with (ICAO, 2012). Some of the 25 rules are presented in Table 2.

The fuzzy risk matrix model was verified in two ways: first, the results were compared with a standard risk matrix as recommended by ICAO (for selected values); second, verification of the fuzzy inference rules was done in accordance with the method described in (Skorupski, 2014, 2015b). The other local fuzzy models mentioned in this paper were checked in the same way.

3. Serious air traffic incident no. 270/06

3.1. Description of the circumstances of the event

The event took place on September 1, 2006 at Warsaw Chopin Airport and involved the following aircraft: Airbus A320 and Embraer EMB170. The Airbus (A320) crew received instructions from the ground controller to taxi down taxiway A and, subsequently, via taxiway E to taxi to runway threshold RWY 29 (Fig. 6) – solid line.

The Airbus A320 pilot acknowledged the taxiing permission properly but continued taxiing incorrectly – using taxiway A4 straight to the runway – double line. At the same time, on runway RWY 29 Embraer (EMB170) began take-off – arrow. Taxiing down taxiway A4 caused a safety hazard for take-off on runway RWY 29. In this situation the airport control tower (TWR) issued a command to the EMB170 crew to stop. The ground controller (GND) proceeded similarly with aircraft A320. Both crews carried out the command.

3.2. Model for accident probability assessment

The event was qualified in the "serious air traffic incident" category since the collision of aircraft almost took place. In (Skorupski, 2015a), six scenarios leading to the transformation of the incident into an accident were defined.

Scenario 1 (S1). The GND controller after issuing taxi clearance and making sure that the A320 crew properly understood it (which took place in the actual case as the crew correctly repeated the clearance) does not monitor the actual execution

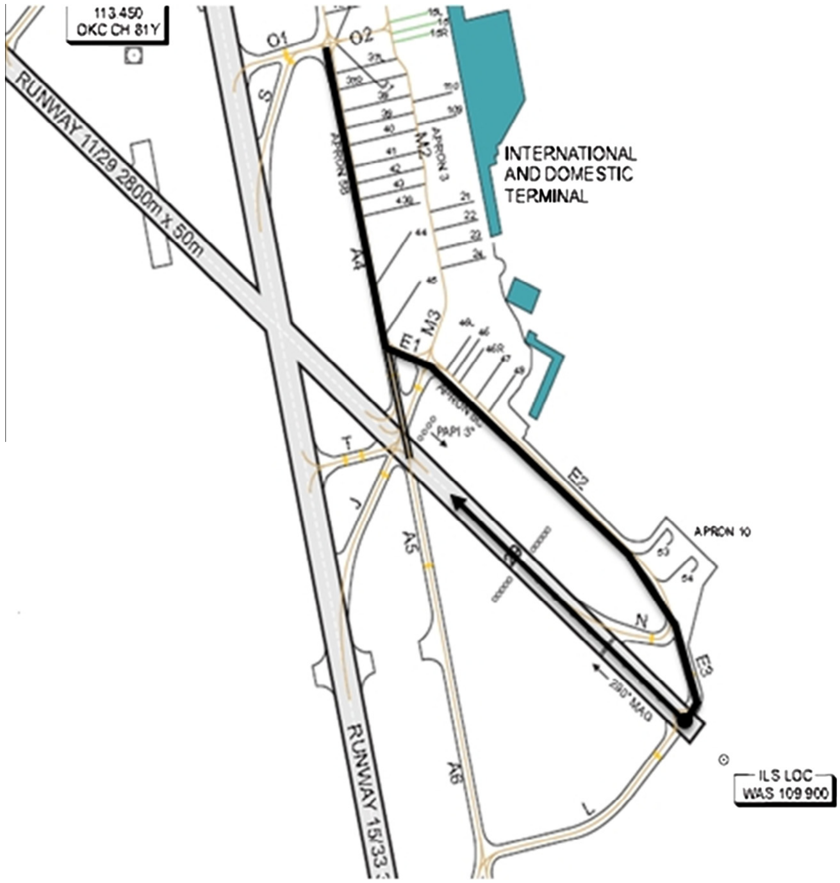

Fig. 6. Scheme of incident no. 270/06.

of this procedure. In fact, this also corresponds to the situation when the GND controller notices incorrect taxiing and issues the command to stop the taxiing, although it will be too late and there will be no opportunity to avoid A320's Runway Incursion.

Scenario 2 (S2). The TWR controller only deals with observation of his/her own area of responsibility and does not pay attention to the taxiway, and therefore does not see the risk in EMB170's take-off. In fact, this also corresponds to the situation when the TWR controller notices incorrect taxiing and issues a command to interrupt the take-off but it will be too late and there will be no possibility to stop the EMB170 aircraft from reaching the collision point.

Scenario 3 (S3). The crew of either aircraft involved in the incident do not respond to the command to stop the actual procedure. In reality, this corresponds to the situation when the crew's reaction time is too long and the aircraft cannot be stopped before the collision point.

Scenario 4 (S4). The weather conditions (visibility) are so poor that it is impossible to observe the actual traffic situation. This applies to both controllers. In the present case the weather conditions were good, but if the same traffic situation had occurred in poor visibility then the probability of an accident instead of an incident would have been much greater.

Scenario 5 (S5). None of the aircraft crews notes a conflict situation. In this particular incident there was the timely intervention of the controllers, so action based on both the pilot's own observation and recognition of the conflict was not necessary. However, in many real-world situations the aircraft crew notices the threat first and undertakes appropriate preventive action. In the accident model, scenario S5 will be represented by the times at which the flight crews notice the danger. If the reaction time is too long, it will not be able to stop the aircraft before the collision point.

Scenario 6 (S6). The time sequence of the events (taxiing and take-off times) is such that the A320 plane makes a runway RWY 29 incursion at precisely the same moment when EMB170, while taking off, passes it, thus causing a crash.

These six scenarios are a kind of general patterns of the events. Most scenarios take into account, for example, the time after which the controllers or pilots notice the danger. This means, in fact, that we are dealing with an infinite number of scenarios. The same general scenario (S1–S6) with different reaction times or different danger notification times will result in different risk assessment. This is because of the different probability of collision or of the different possible consequences of an accident. The latter are also not determined precisely but in a fuzzy way, with the many intermediate values they can take.

As can be observed, in scenarios S1–S3 and S5 it is most important if it is possible to stop the aircraft before the collision point. These are binary events – either the aircraft stops or it does not stop. However, the process that determines this is dynamic. Both the response time of the individual controllers and of the crews but also the dynamic characteristics (speed, accelerations and decelerations) of the aircraft movements must be considered. The model takes into account all of these dynamic elements, but the end result is the logical (binary) event – the conflict point is either occupied or not. Scenario S4 will not be assessed probabilistically, as this analysis makes sense in a general risk assessment, it does not make sense, however, in the case of studying a particular incident in which the weather conditions are known. However, the case of insufficient visibility is possible in reality and has a significant impact on the final result of traffic occurrence. The model in scenario S4 assumes that the conditions are either sufficient or insufficient – without probabilistic analysis. Scenario 6 requires an analysis of the time dependencies between the planes involved in the incident. We denote by X_1 a random variable describing the time at which A320 enters the runway, while by X_2 a random variable indicating the time at which the EMB 170 aircraft reaches the intersection of runway 29 with taxiway A4. The condition for an accident to occur is to implement scenario 6. Scenarios 1–5 modify the probability of scenario 6. Of course, we should consider not so much the equality of X_1 and X_2 but rather if the difference between them is less than a given limit.

A model in the form of a hierarchical Petri net was developed in order to determine the probability of an accident occurrence. It consists of five pages:

1. *Traffic* – contains the initialization part and a fragment of the model mapping the situation when both planes continue their movements.
2. *Weather* – responsible for checking the weather conditions.
3. *GND Monitor* – responsible for mapping the activities of the GND controller and the crew of the taxiing A320 aircraft.
4. *TWR Monitor* – responsible for mapping the corresponding actions of the TWR controller and the crew of the EMB170 aircraft that is taking off.
5. *EMB stop* – responsible for verifying the possibility of stopping the aircraft before the collision point in case the conflict is recognized and the decision is made to begin the rejected take-off procedure.

A part of the model containing the *TWR Monitor* page is presented as an example (Fig. 7).

The simulation method was used to determine the probability of the incident transforming into an accident. This means that a series of experiments is conducted using the model and the results are observed. These results are treated as a random sample for a group of experiments. In this case, these experiments can be equated with multiple executions of an identical serious incident and observing how often it transforms into an accident.

Simulation analysis requires determining the time necessary to execute different events as well as the probabilities of a variety of actions performed by the pilots and the controllers. In the model of the accident they are treated as random variables. These variables are determined by the time necessary for the crew to recognize the conflict or to react to the warning coming from the outside. Analyses of the problems of human perceptual capabilities and the reaction time fall within the area of psychology and are beyond the scope of this work. However, in order to illustrate the proposed method these values have been estimated by experts – an experienced air traffic controller and the pilot of a military transportation aircraft. The experts identified the expected values of these times. It was assumed that these random variables are described by exponential distribution. However, one should bear in mind that these are largely expert estimates and further research should be carried out in this area, including with the use of formal methods (Kurowicka et al., 2008; Hanea et al., 2010). Data derived from the author's own measurements of the aircraft taxiing process at Warsaw Chopin Airport were also used. The measurements were completed from May to November of 2012.

The model was verified at three stages. First, the results from the model were compared with the actual course of the incident. Second, a theoretical analysis of the reachability graph for the Petri net model was conducted. In order to prove the correctness of the model it is necessary to check whether for all dead states (final markings) it is possible to determine if there has been a transformation of an incident into an accident. Third, verification was done using expert knowledge. The experts mentioned above were asked to confirm the correctness of the results.

4. Analysis of incident no. 270/06 with the use of fuzzy logic

4.1. Model of severity of the consequences

The consequences in the case of continuing air traffic event no. 270/06 depend on the behavior of both crews. In the situation, such as in the factual event, collision did not occur. However, providing that A320 had at least partially remained on the runway and EMB170 did not interrupt take-off, then the consequences could even have been disastrous. An analysis of the severity of the consequences will therefore be based on the following variables:

Input variables:

1. *A320 behavior* – a linguistic variable taking the following values: *braking*, *maneuver*, *none*. In the case of a situation when the distance between the aircraft and the point of collision is large at

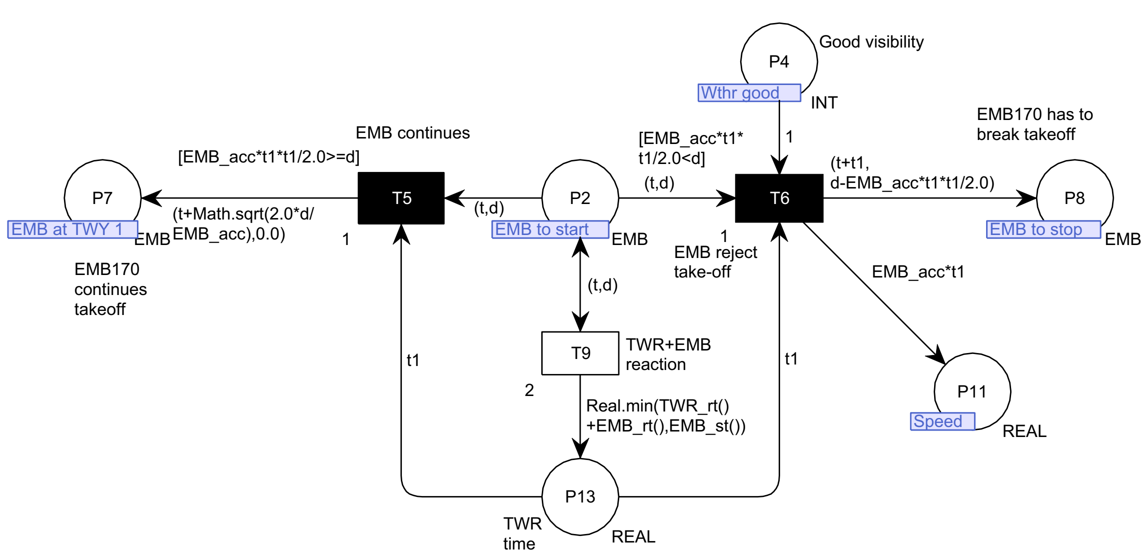

Fig. 7. *TWR Monitor* page of the model of the accident raised from serious incident 270/06.

the moment when the pilot detects the hazard, the *A320 behavior* variable will take the value *braking*. In the case of a situation when this distance is insufficient for total stopping, this variable takes the value *maneuver*. This means that the crew will reduce the velocity of the aircraft and will perform an avoidance maneuver in close proximity to the point of collision. In the case of a situation when neither braking nor avoidance maneuvers are possible at the moment when the pilot detects the danger, then variable *A320 behavior* takes the value *none*. The delimitation between the particular values of this linguistic variable is not strict; therefore, they are represented by fuzzy sets whose membership functions are shown in Fig. 8.

2. *EMB170 behavior* – a linguistic variable taking the following values: *braking*, *maneuver*, *none*. The form of the membership functions and their sense is similar to variable *A320 behavior* (Fig. 8).

The output variable is *Severity of consequences*. It takes the fuzzy values described in Section 2.2 presenting the model of risk assessment. Determination of these values was based on fuzzy inference rules coming from the experts. These are presented in Table 3.

It must be noted that all items in Table 3 are fuzzy values. This means that the rules can be met to a different extent, e.g. in the situation described by rule 6 we can expect a collision with catastrophic consequences. However, it is also possible that the accident may be closely avoided. This may happen when either rule 4 or 5 is fulfilled at least partially. This means a case when any of the pilots at least makes an attempt to perform an avoidance maneuver. Additionally, for each situation the probability of an accident is calculated and taken into account in the risk assessment.

4.2. A320 behavior model

Actions undertaken by the A320 crew depend mainly on previously planned actions and on the GND controller's reaction timing. According to the available incident investigation report we cannot determine the real cause of passing the correct taxiway E1 and continuing taxiing via A4. There are several possible scenarios, among which two are the most probable:

- Either the aircraft pilot understood the taxiing instruction incorrectly and planned to taxi to taxiway threshold RWY29 via taxiways A5, A6 and L
- or the aircraft pilot understood the instruction correctly but missed the taxiways due to bad marking and poor weather conditions.

In the former case one should expect that the crew will be focused on observing the airfield. This variant of taxiing requires crossing the runway and one should suppose that the awareness of the crew will make them carefully watch any turn of events.

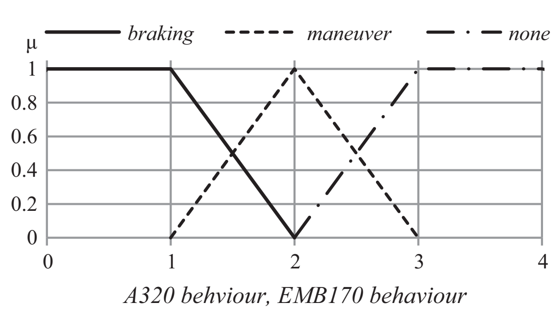

Fig. 8. Membership functions of the linguistic input variables *A320 behavior* and *EMB170 behavior*.

Table 3
Fuzzy inference rules for the *Severity of consequences* variable.

Rule number	*A320 behavior*	*EMB170 behavior*	*Severity of consequences*
1	*braking*	any	*minor*
2	any	*braking*	*minor*
3	*maneuver*	*maneuver*	*major*
4	*maneuver*	*none*	*hazardous*
5	*none*	*maneuver*	*hazardous*
6	*none*	*none*	*catastrophic*

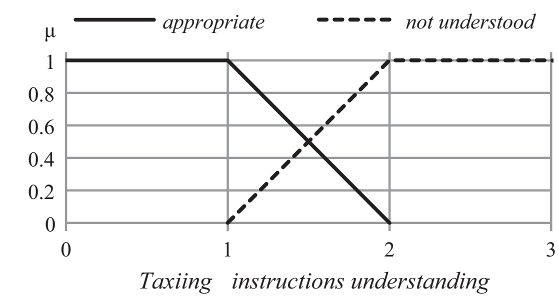

Fig. 9. Membership functions of the linguistic input variable *Taxiing instructions understanding*.

In such a case the probability of total stopping or, alternatively, of performing an avoidance maneuver is high. On the other hand, such a possibility is weakened by the fact that the GND controller issuing permission for taxiing did not mention the necessity to hold short of the runway (which is obvious as the original taxiing permission did not take into consideration crossing the runway at all).

In the latter case, the actions performed by the aircraft pilot depend on his/her knowledge of the airport's topography. Providing that the aircraft pilot knows it well, one can assume that knowing that the taxiway does not cross the runway, he/she will hold short of it and an unauthorized intrusion onto the runway will not occur. In the opposite situation, the lack of any reaction or performing the avoidance maneuver at the last possible moment is more probable. A separate issue affecting the actions performed by the A320 aircraft pilot is knowledge about any construction work that should come from the NOTAMs and also the validity of the Jeppesen maps. Yet another possible cause of error are inappropriate taxiway markings and poor weather conditions preventing proper detection of the right taxiway. All of these parameters affect the time necessary for the A320 crew to recognize a hazardous situation.

The input linguistic variables are:

1. *Taxiing procedure understanding* – a linguistic variable taking the following values: *appropriate*, *not understood*.
2. *A320 recognition of conflict* – a linguistic variable taking the following values: *short*, *average*, *long*. These represent the time after which the A320 crew realizes that their taxiing is incorrect.
3. *GND controller recognition of conflict* – a linguistic variable taking the following values: *short*, *average*, *long*. These represent the time after which the GND controller realizes that the taxiing of A320 is incorrect and reacts by commanding it to stop. This situation will also be influenced by the weather conditions and other difficulties of observation (construction work).

The values of the input linguistic variables are presented in Figs. 9–11. The output variable *A320 behavior* takes fuzzy values defined in the same way as in the *Severity of consequences* model.

The model is complemented by the fuzzy rules base obtained from the experts, i.e. the air traffic controllers and aircraft pilots. There are 13 defined rules, some of them are presented in Table 4.

4.3. EMB170 behavior model

In the analyzed case, actions performed by the EMB170 crew were correct – the instruction to discontinue take-off was properly understood and properly (i.e. quickly enough) performed. However, one cannot exclude the situation that the rejected take-off procedure would begin after a long period of time or would not occur at all. Such cases change not only the probability of an accident but can also affect its consequences. In the case of the implementation of a braking procedure, the aircraft will reduce its velocity and, as a result, the force of the potential impact will be lower. Subsequently, the probability of a successful avoidance maneuver will increase. Otherwise, collision will occur while the EMB170 aircraft will keep increasing its velocity – then the consequences will be catastrophic. The behavior of the EMB170 crew will depend on the following input variables:

1. *TWR controller recognition of conflict* – a linguistic variable representing the time after which the TWR controller will recognize the danger and will provide the instruction for the EMB170 crew. This time period also includes the time necessary for transmission, e.g. resulting from the necessity of repeating the command. This variable can take the following values: *short*, *average*, *long*.
2. *EMB170 recognition of conflict* – a linguistic variable describing the crew's ability to recognize the danger and to perform a proper (quick) reaction. This variable takes into account: situational awareness, experience, and teamwork abilities as well as visibility conditions. All of these factors determine the time necessary to begin the rejected take-off procedure. It can take the following values: *short*, *average*, *long*.
3. *Braking conditions* – a linguistic variable describing the runway surface condition which has an impact on the coefficient of friction and therefore on the effectiveness of the braking maneuver. This variable can take the following values: *good*, *medium*, *poor*. The basis for determining the value of this linguistic variable is the assessment (or measurement) of the coefficient of friction on the runway.

The membership functions of the input linguistic variables are presented in Figs. 12–14.

The output variable *EMB170 behavior* takes fuzzy values, as was described in Section 4.1. Again, the values of this variable are obtained from a fuzzy inference system based on the fuzzy knowledge base obtained from the experts. There are 21 defined rules; some of them are presented in Table 5.

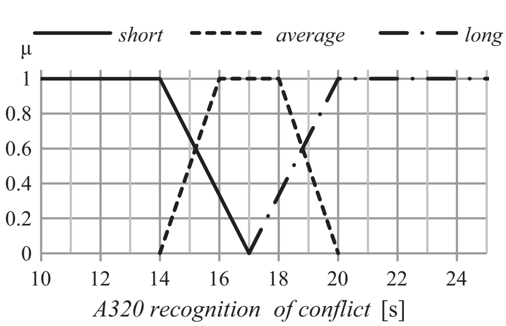

Fig. 10. Membership functions of the linguistic input variable *A320 recognition of conflict*.

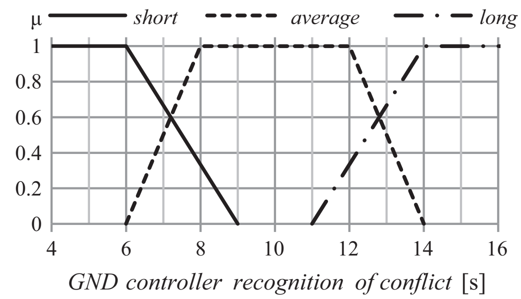

Fig. 11. Membership functions of the linguistic input variable *GND controller recognition of conflict*.

Table 4
Fuzzy inference rules for linguistic variable *A320 behavior*.

Rule number	*Taxiing instructions understanding*	*A320 recognition of conflict*	*GND controller recognition of conflict*	*A320 behavior*
2	*appropriate*	*average*	*short*	*braking*
4	*appropriate*	*average*	*long*	*none*
5	*appropriate*	*long*	*short*	*maneuver*
10	*not understood*	*average*	*average*	*braking*
11	*not understood*	*average*	*long*	*maneuver*
13	*not understood*	*long*	*long*	*none*

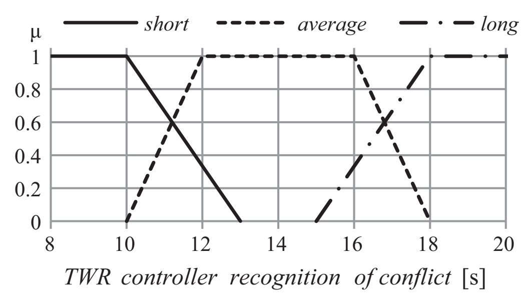

Fig. 12. Membership functions of the input linguistic variable *TWR controller recognition of conflict*.

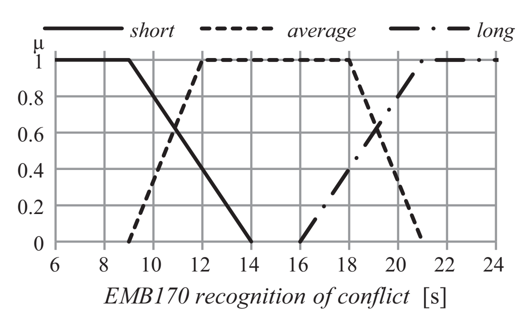

Fig. 13. Membership functions of the input linguistic variable *EMB170 recognition of conflict*.

4.4. Probability of event estimation

As was mentioned above, the probabilities of the particular traffic situations will be estimated by using a model in the form of a Petri net as described in (Skorupski, 2015a). This model was complemented and provided with details necessary to achieve the goals of this paper. This applies, for instance, to taking into account the different values of the coefficient of friction during braking which have an impact on the effectiveness of emergency braking

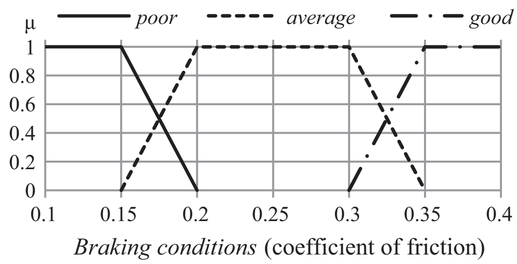

Fig. 14. Membership functions of the input linguistic variable *Braking conditions*.

Table 5
Fuzzy inference rules for the *EMB170 behavior* variable.

Rule number	*TWR controller recognition of conflict*	*EMB170 recognition of conflict*	*Braking conditions*	*EMB170 behavior*
3	any	*short*	*poor*	*maneuver*
5	*short*	*average*	*average*	*braking*
14	*average*	*long*	*average*	*braking*
17	*average*	*long*	*average*	*none*
19	*long*	*long*	*good*	*maneuver*
20	*long*	*long*	*average*	*none*

when executing a rejected take-off procedure. Additionally, the factors described in Sections 4.2 and 4.3 were taken into account when estimating the pilots' reaction times on the controllers' instructions and also the time necessary for them to notice the danger on their own.

5. Computer tool and simulation experiments

The hierarchical fuzzy inference system, developed to estimate the severity of the consequences and, finally, the risk of an accident, was implemented in the SciLab 5.4 environment with a Fuzzy Logic Toolbox add-on.

The model of the dynamics of the incident developed as a Petri net and the risk assessment model implemented as a fuzzy risk matrix are two independent models with independent computer implementations. The former is in the CPN Tools 4.0 environment, the latter is in the SciLab 5.4 environment as FMRE (Fuzzy Matrix Risk Evaluation) software. Both tools are used in the following way:

1. The input parameters are determined depending on the type of experiment.
2. A series of simulation runs is conducted using the model implemented as a Petri net; the result is the probability of a transformation of an incident into an accident.
3. The risk is assessed using the fuzzy risk matrix (FMRE software); the result of point 2 is one of the inputs to the model.

Simulation experiments were conducted using the following plan: the first experiment consisted in estimating the risk for a situation identical to a real, serious incident. The next step was an analysis of how the risk would change in the situation when the aircraft crews reacted within a longer period of time due to lower qualifications, worse teamwork skills or, eventually, due to lower situational awareness. As the third simulation experiment an analysis was conducted of how the risk would change in the case of the occurrence of accident scenarios (Section 3.2) or their combinations. Within the fourth experiment an analysis of the influence of meteorological conditions was extended with the problem of different braking conditions expressed by the change of the runway coefficient of friction. The fifth experiment consisted in risk assessment in the case when all of the aircraft crews' reaction times grew by a certain amount of time.

In each experiment the input values were estimated by the experts and a modified version of the model from (Skorupski, 2015a) was used. The times required for both the aircraft crews and controllers to notice the danger as well as the times needed to react to the instructions given by the air traffic controller that were used in the model were treated as random variables. Their average values are shown in the tables presenting the results of the analyses. It should be noted that the probability estimations do not apply to the absolute probability of an accident but to the probability of the evolution of an incident that already occurred into an accident.

By using FMRE software one can assess the severity of the consequences and, finally, the risk by using a fuzzy risk matrix. In the case of risk assessment the results apply to the transformation of an incident that occurred into that of an accident.

5.1. Nominal situation – as in the incident

In this section the risk of an event will be assessed providing that all of the parameters take values that are the same as in the real incident. These parameters are presented in Table 6. The source of the data is a report from the incident investigation (Civil Aviation Authority, 2008). Unfortunately, some of the parameters cannot be unequivocally determined by using this report. This applies, for example, to the issue of understanding the taxiing instruction that was given to the A320 pilot. The report states only the fact of incorrect taxiing but does not specify its cause. It is also worth mentioning that this is a common issue – the reports of "less important" events (incidents and serious incidents) are not as detailed as the reports of accidents. Therefore, some of the elements were adopted in the same way as in (Skorupski, 2015a) to make the results comparable.

Applying the local fuzzy inference system *A320 behavior* allows us to estimate this parameter at the level of 0.77, which corresponds to the linguistic variable *braking*. An identical result was obtained for the *EMB170 behavior* model. This is consistent with the course of the real incident. Taking this into account, as a result of applying the local model *Severity of consequences* we receive a value of 1.25 which corresponds to the linguistic variable *minor*. The consequences that may occur in the case of successful emergency braking (without a collision) are, e.g. damage of the braking system or tires, or minor injuries of the passengers caused, for example, by being hit by small objects that were not properly secured for the duration of the take-off. The probability of an accident as received from the Petri net model equals, for this example,

Table 6
Risk assessment – parameters as in serious incident no. 270/06.

Input parameter	Value	Output parameter	Value
Taxiing instructions understanding	*not understood*	*A320 behavior*	0.77 (*braking*)
A320 recognition of conflict	15 [sec]		
GND controller recognition of conflict	10 [sec]		
Braking conditions (coefficient of friction)	0.25	*EMB170 behavior*	0.77 (*braking*)
EMB170 recognition of conflict	15 [sec]		
TWR controller recognition of conflict	10 [sec]		
A320 behavior	0.77	*Severity of consequences*	1.25 (*minor*)
EMB170 behavior	0.77		
Probability of an accident	$10^{-3.28}$	*Risk tolerance*	2.36 (*tolerable*)
Severity of consequences	1.25		

Table 7
Risk assessment – the pilots and controllers notice the hazardous situation early.

Input parameter	Value	Output parameter	Value
Probability of an accident	$10^{-5.49}$	*Risk tolerance*	1.75 (*tolerable/ acceptable*)
Severity of consequences	1.25		

Table 8
Risk assessment – the pilots and controllers notice the hazardous situation late.

Input parameter	Value	Output parameter	Value
Taxiing instructions understanding	*not understood*	*A320 behavior*	3.23 (*none*)
A320 recognition of conflict	30 [sec]		
GND controller recognition of conflict	20 [sec]		
Braking conditions (coefficient of friction)	0.25	*EMB170 behavior*	3.23 (*none*)
EMB170 recognition of conflict	30 [sec]		
TWR controller recognition of conflict	20 [sec]		
A320 behavior	3.23	*Severity of consequences*	3.97 (*catastrophic*)
EMB170 behavior	3.23		
Probability of an accident	$10^{-1.95}$	*Risk tolerance*	4.24 (*intolerable*)
Severity of consequences	3.97		

$5.26 \cdot 10^{-4}$. After transformation into the adopted logarithmic scale it gives a value of -3.28, which can be expressed by the linguistic variables as a value between *high* and *medium*. The final risk assessment takes the value of 2.36 (ranging from 0 to 5 in the adopted scale), which corresponds to a *tolerable* value.

5.2. Analysis for a more or less efficient crew

This section presents an assessment of the risk in circumstances that are similar to the analyzed incident but in the case when the ability to perceive the hazard and the resulting crews' reaction times vary from the values that took place in the real event.

First, the situation in which both the crews and the controllers needed half as much time to notice a hazardous situation as in the real incident was analyzed. Additionally, in the model used to determine the probability of a transformation from an incident into an accident for both crews, half as long a reaction time to the instruction received from the controller to stop the operation was included. The final results of the analysis are presented in Table 7.

Already at the nominal conditions of the analyzed incident, the reactions of both of the aircraft crews were appropriate and sufficiently fast to avoid an accident; also after shortening all of the response times, which this experiment confirms. The determined probability is significantly lower than previously and equals $10^{-5.49}$. The severity of the consequences is unaffected, as it is tied to the same fact of emergency braking. The final risk assessment takes the defuzzified value of 1.75, which can be expressed by the linguistic variables as a value between *tolerable* and *acceptable*. This means a lower risk and can indicate a potential course of action within the risk management system. A reduction of the reaction time considered in this experiment can be achieved, for example, by appropriate simulator training.

Second, an experiment was conducted consisting in risk assessment in the case when both the crews and the controllers react significantly slower than in the real incident. In the model for determining the probability of transformation from an incident into an accident, twice as long a reaction time to the controller's instruction to break the operation was included. Additionally, twice as long a time necessary to transmit this instruction by radio was included. The results of this experiment are presented in Table 8.

The results of this experiment show that this event was properly interpreted as a serious air traffic incident, which means that an accident almost happened. In the case of longer reaction times the crews of both aircraft do not take any action, and therefore the occurrence of an accident depends only on the difference in time between the moments of occupying a collision point by the aircraft. In such a situation the severity of the consequences can be estimated as *catastrophic*, and the probability of an accident is very high and equals $10^{-1.95}$. Obviously, in such a case the risk is significantly higher and in the adopted scale it takes the defuzzified value of 4.24. This can be expressed in the linguistic sense by the *intolerable* value.

Longer times necessary to recognize the hazardous situation and longer reaction times could of course occur in reality. The most probable reason for such a situation could be the low visibility that would hinder recognition of the situation for all of the participants – the aircraft crews and the controllers. Additionally, the A320 crew was taxiing improperly. This means that the crew's situational awareness was poor and probably teamwork among the crew was not proper as well. Such a situation could take place, for example, due to concentrating too much on operating onboard systems or other tasks, such as planning the further flight. All of these factors do not favor observation of the airfield. They also increase the probability of remaining unaware of a hazardous situation, even in case of the proper reaction of the GND controller.

5.3. Analysis for accident scenarios

In Section 3.2 several scenarios were presented whose occurrence could cause a real, serious incident to transform into an air traffic accident. In this section, risk assessment for those scenarios and their combinations will be presented. The results were obtained from an experiment based on the use of the proposed models. The most important results are presented in Table 9.

Implementation of each scenario (with the exception of scenario S4) leads to the *tolerable* value. However, by using the fuzzy risk matrix we can distinguish between the ratings. This could be useful in prioritizing actions leading to an elimination of potential threats. In the case of scenario S4, which corresponds to extremely poor weather conditions, and especially the lack of visibility, the assessment of risk shows that it is at an extremely high level (4.24 which corresponds to the value of *intolerable*). This means

Table 9
Risk assessment for accident scenarios.

Scenario	*Severity of consequences*	Probability of an accident	*Risk tolerance*
S1	1.25	$10^{-2.78}$	2.43 (*tolerable*)
S2	1.25	$10^{-2.61}$	2.47 (*tolerable*)
S3 (A320)	1.25	$10^{-2.26}$	2.56 (*tolerable*)
S3 (EMB170)	1.25	$10^{-1.94}$	2.64 (*tolerable*)
S4	3.97	$10^{-0.03}$	4.24 (*intolerable*)
S5	1.25	$10^{-2.09}$	2.61 (*tolerable*)
S2 + S5	3.25	$10^{-1.33}$	4.01 (*intolerable*)

Table 10
Risk assessment – snow–water mixture covering the runway.

Input parameter	Value	Output parameter	Value
Taxiing instructions understanding	*not understood*	*A320 behavior*	2.4 (*maneuver/none*)
A320 recognition of conflict	18 [sec]		
GND controller recognition of conflict	14 [sec]		
Braking conditions (coefficient of friction)	0.12	*EMB170 behavior*	2.0 (*maneuver*)
EMB170 recognition of conflict	15 [sec]		
TWR controller recognition of conflict	10 [sec]		
A320 behavior	2.4	*Severity of consequences*	2.62 (*major*)
EMB170 behavior	2.0		
Probability of an accident	$10^{-2.99}$	*Risk tolerance*	2.85 (*tolerable*)
Severity of consequences	2.62		

that it is necessary to implement special procedures for risk reduction. The most common among them are low visibility procedures (LVP). The obtained results confirm its significance and indicate the necessity to design them with utmost accuracy. Further research is required to answer the question as to at which boundary visibility such procedures should be introduced.

Among the combinations of two scenarios, special attention should be paid to the situation when scenarios S2 and S5 occur simultaneously. This is the case when both aircraft do not notice the danger combined with the lack of proper reaction of the TWR controller. The risk is therefore at the level of *intolerable*. Such a result shows the specific role of the TWR controller in preventing air traffic accidents. If he/she is the only one that behaves improperly, the risk of an accident is relatively low, but if combined with other mistakes the risk increases significantly.

5.4. The case of poor weather conditions

Within the analysis of accident scenarios the case of extremely poor visibility conditions at the airport was analyzed. This section will extend the analysis of the influence of weather conditions by adding the question of the runway surface condition. Risk will be assessed for the case of the simultaneous occurrence of intense rain and a snow–water mixture covering the runway. The coefficient of friction for such conditions was derived from (Raymer, 1992). Additionally, a slight increase in the time necessary to notice a hazardous situation by participants related to the A320 aircraft, i.e. its crew and the GND controller, was taken into consideration. The results are presented in Table 10.

As is shown, the deterioration of the braking conditions by lowering the coefficient of friction causes the EMB170 aircraft which is taking off not be capable of stopping before reaching the collision point and, therefore, making an avoidance maneuver is necessary. On the other hand, a longer reaction time of the A320 aircraft crew means their reaction will either be an avoidance maneuver or there will be no reaction at all. This places the estimation of consequences at the level of *major*. Combined with the calculated probability of an accident, this places the risk assessment at the level of *tolerable*. However, applying a fuzzy risk matrix can clearly indicate a shift towards the *intolerable* value.

5.5. Dependence on the crew reaction time

The results of the experiments presented in the previous sections show a significant influence of the dynamics of events on risk assessment. The dynamics is expressed by the reaction time of the participants of the incident. In this section the results of the analysis will be presented providing that:

- the crews of both aircraft notice the danger with a slight delay t_o
- the TWR and GND controllers notice the danger within the time period as in a real incident
- the crews of both aircraft react slower than in the nominal conditions to the commands given by the controllers to interrupt the procedure currently being executed. This delay is also equal to t_o.

The results of this experiment are presented in Table 11.

It is worth mentioning that the results of the previous experiments show that if only one of the participants notices the danger or reacts too late this does not significantly affect the assessment of the risk. The proper reactions of the other participants allow to keep the risk at the *tolerable* level. The results of this experiment show instead that the simultaneous elongation of the crew's reaction time to the controllers' commands and noticing the danger by both of the crews all have a much more significant influence. It can be observed that 4 seconds' delay of the reaction time of both crews shifts the risk to the *unacceptable* area, while the situation that everyone will react too slowly is unlikely to occur, it is clear that the time limit in such a situation is very narrow.

6. Discussion and conclusions

In this paper we presented the concept of a fuzzy risk matrix applied to air traffic accident analysis. It allows to express both the probability of an event and its consequences in the form of fuzzy linguistic variables. On this basis a fuzzy inference system can be created that can be used to assess risk in a continuous scale, therefore giving the possibility to compare different cases, even when they belong to the same risk category. This is very important if we want to effectively manage risk in air transport. As the results of the experiments presented in Section 5 show, situations in which the risk can be classified as *tolerable* may, in fact, differ significantly from one another. Showing that certain actions either increase or decrease the risk as well as the quantification of the scale of change can constitute valuable support for safety managers.

The method presented here is an example of the proactive approach to risk management which is so often postulated. Typically we look for the causes of events and then wonder what safety barriers to implement to prevent the same accident from happening again. This paper proposes a different approach in that we analyze the incident by searching for the safety barriers that caused that there *was no accident*, then we analyze the scenarios under which these barriers do not work. This allows to proactively eliminate such scenarios. Quantitative assessments of risk at the *tolerable* level when the crew has good situational awareness and very quickly notices the danger were obtained. However, the risk can

Table 11
Dependence of the risk on reaction time elongation of the aircraft crews.

Elongation of crew reaction time t_o [s]	1	2	3	4	5	6	7
Probability of an accident	$10^{-3.11}$	$10^{-2.93}$	$10^{-2.81}$	$10^{-2.67}$	$10^{-2.55}$	$10^{-2.44}$	$10^{-2.33}$
Severity of consequences	1.25	1.25	2.24	3.69	3.97	3.97	3.97
Risk tolerance	2.36	2.39	2.72	3.94	3.99	4.04	4.08

reach an *intolerable* level in case of a barely trained crew that is not well oriented in the traffic situation. The influence of weather conditions (visibility, runway coefficient of friction) on the risk of the transformation from an incident into an accident was also examined. Similarly, the defined accident scenarios were compared up to what extent they increased the risk of an accident. Scenario S4 and the simultaneous occurrence of scenarios S2 and S5 were identified as the most dangerous ones requiring special attention from airport safety management units. It was also demonstrated that the simultaneous delay in the reaction time of several participants of an incident has a significant influence on the deterioration of risk assessment. The categorization of scenarios in terms of increased risk allows to take appropriate actions in the most vulnerable places.

The experiments conducted here allow to conclude that a fuzzy risk matrix is a convenient tool to assess risk in air transport. The same approach can be applied to other modes of transport as well as in other areas. An important advantage is that the continuous scale of risk replaces the discrete one.

Further research should focus on creating an integrated tool to assess both the probability and consequences of the considered events. Further research is also needed in order to determine the probabilistic characteristics of the adopted random variables. Yet another important area requiring research is to determine the boundary conditions (e.g. visibility) at which the risk increases up to such a level that conducting special procedures, such as low visibility procedures (LVP), is necessary.

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